

5. To find the method of moments estimators for  $\lambda$  and  $\theta$ , we must solve the system of equations

$$\mathbb{E}(Y) = \bar{Y} \quad \text{and} \quad \mathbb{E}(Y^2) = \frac{1}{n} \sum_{i=1}^n Y_i^2$$

Since  $\mathbb{E}(Y) = \theta$  and  $\text{Var } Y = 2\lambda^{-2}$ , we find  $\mathbb{E}(Y^2) = \text{Var } Y + [\mathbb{E}(Y)]^2 = 2\lambda^{-2} + \theta^2$ . Thus,

$$\theta = \bar{Y} \quad \text{and} \quad 2\lambda^{-2} + \theta^2 = \frac{1}{n} \sum_{i=1}^n Y_i^2$$

and so some trivial algebra gives

$$\hat{\theta}_{\text{MOM}} = \bar{Y} \quad \text{and} \quad \hat{\lambda}_{\text{MOM}} = \sqrt{\frac{2n}{\sum_{i=1}^n Y_i^2 - n\bar{Y}^2}}.$$