Stat 252 Winter 2007
Assignment #9

This assignment is due at the beginning of class on Wednesday, April 4, 2007. You must submit all problems that are marked with an asterix (*).

1. * Let $Y \sim \text{Exp}(\theta)$. Determine the rejection region of the most powerful significance level α test of $H_0 : \theta = 1$ against $H_A : \theta = 1/2$. (Hint: Construct the likelihood ratio test.)

2. * Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed with density function

$$f(y|\theta) = \theta \exp(-\theta y)$$

where $y > 0$ and $\theta > 0$. As usual, let $\overline{Y}$ denote the sample mean given by

$$\overline{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i.$$

It is known that the maximum likelihood estimator of θ is $\hat{\theta}_{\text{MLE}} = 1 / \overline{Y}$.

(a) Consider testing $H_0 : \theta = \theta_0$ against $H_A : \theta \neq \theta_0$. Verify that the rejection region for the generalized likelihood ratio test of these hypotheses is of the form

$$\left\{ \overline{Y} \exp(-\theta_0 \overline{Y}) \leq C \right\}$$

for some suitable constant C.

(b) Suppose, to be specific, that $\theta_0 = 1$, and that a random sample of size $n = 10$ is conducted. If the observed data yield $\overline{Y} = 1.25$, perform the generalized likelihood ratio test at the approximate significance level $\alpha = 0.10$.

3. * Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed with density function

$$f(y|\theta) = \theta^2 \, y \exp(-\theta y)$$

where $y > 0$ and $\theta > 0$. As usual, let $\overline{Y}$ denote the sample mean given by

$$\overline{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i.$$

It is known that the maximum likelihood estimator of θ is $\hat{\theta}_{\text{MLE}} = 2 / \overline{Y}$.

(a) Consider testing $H_0 : \theta = 1$ against $H_A : \theta \neq 1$. Verify that the generalized likelihood ratio for this hypothesis testing problem is

$$\Lambda = \left(\frac{\overline{Y}}{2} \right)^{2n} \exp(2n - n\overline{Y}).$$

(b) Suppose that a random sample of size $n = 5$ is conducted and the observed data yield $\overline{Y} = 1.0$. Perform the generalized likelihood ratio test at the approximate significance level $\alpha = 0.05$.