Stat 252 Winter 2007
Assignment #2

This assignment is due at the beginning of class on Monday, January 22, 2007. You must submit all problems that are marked with an asterix (*).

1. Show that if $Y_1, Y_2, \ldots, Y_n$ is a random sample from a population with mean μ and variance σ^2, then

$$S^2 := \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$$

is an unbiased estimator of σ^2. This illustrates one reason for dividing by $n-1$ in the definition of the sample variance S^2, instead of dividing by (the more natural) n.

2. * A medicinal herb growing operation maintains a generator to power 25 heat lamps in its greenhouse so that when one lamp fails, another immediately takes over. (Only one lamp is lit at a time.) The heat lamps operate independently, and each has a lifetime which is normally distributed as $\mathcal{N}(50, 4)$ (in hours). If the greenhouse is not checked for 1300 hours after the generator is turned on, what is the probability that a lamp will be burning at the end of the 1300-hour period?

3. * Suppose that $Y_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ are independent. Using moment generating functions, show that the distribution of the statistic $Y_1 + Y_2$ is $\mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

4. * Each of the following two problems can be solved using an appropriate theorem and definition from the lecture of January 15, 2007.

(a) Suppose that $Y_1, Y_2, \ldots, Y_n$ are i.i.d. normal random variables with common mean μ and common variance σ^2. Show that the random variable

$$\frac{\overline{Y} - \mu}{S/\sqrt{n}}$$

has a t distribution with $n-1$ degrees of freedom.

(b) Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. normal random variables with common mean μ_1 and common variance σ_1^2. Suppose further that $Y_1, Y_2, \ldots, Y_m$ are i.i.d. normal random variables with common mean μ_2 and common variance σ_2^2, and that the X_i are independent of the Y_j. If

$$S_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \quad \text{and} \quad S_2^2 = \frac{1}{m-1}\sum_{i=1}^{m}(Y_i - \overline{Y})^2$$

then show that

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

has an F distribution with $(n-1)$ numerator degrees of freedom, and $(m-1)$ denominator degrees of freedom.

(continued)

5. Do the following exercises from Wackerly, et al.

- page 351 #7.38
- page 352 #7.40
- page 352 #7.41

6. Do the following exercises from Wackerly, et al.

- page 323 #6.64
- page 324 #6.65