Statistics 252 "Practice Exam" – Winter 2006

Note that all of these problems appeared on previous exams, and are relevant for this semester's course. However, completing this "practice exam" should constitute only a portion of your study and preparation for the midterm.

1. Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables. It is known that $\sigma^2 = 9$, but μ is unknown. You want to test $H_0 : \mu = 0$ against $H_A : \mu > 0$ by rejecting H_0 when $\overline{Y} > c$.

- **(a)** Find c so that this test has significance level $\alpha = 0.05$. (Of course, c will depend on the sample size n.)
- **(b)** Using the test determined in **(a)**, find the power of the test when $\mu = 1$ and $n = 36$.
- **(c)** Again, using the test determined in **(a)**, show that the power when $\mu = 1$ will increase as the sample size increases. Regardless of whether or not you can show this result, does it make sense intuitively? Comment **very** briefly.

2. Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables. It is known that $\sigma^2 = 4$, but μ is unknown. You want to test $H_0 : \mu = 0$ against $H_A : \mu > 0$ by rejecting H_0 when $\overline{Y} > 3.92/\sqrt{n}$.

- **(a)** Verify that this test has significance level $\alpha = 0.025$.
- **(b)** Say you are planning an experiment for which the data will be analyzed by this hypothesis test. How large a sample should you collect if you would like the test to have power 0.9 when $\mu = 0.5$?

(*NOTE: Think through this problem. Do not look up formulæ we did not cover in class.*)

3. John carries out hypothesis tests using 0.01 as the significance level, while George uses 0.05 as his significance level. Ringo carries out an experiment to compare two hypotheses, and computes the p-value. But all Ringo tells John and George is that the p-value is smaller than 0.03. Can John make his decision to accept or reject the null? If so, what is the decision? What about George? Justify your answers **very** briefly.

4. In the context of Stat 252, clearly define what is meant by a *significance level* α *hypothesis test.*

5. Let $Y_1, \ldots, Y_{100}$ be independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables. It is known that $\sigma^2 = 25$, but μ is unknown. Consider testing $H_0 : \mu = 0$ against $H_A : \mu < 0$ by rejecting H_0 for small values of the sample mean $\overline{Y}$. The user of the test would like to treat the hypotheses somewhat symmetrically. In particular, she wants the probability of Type I error to be equal to the probability of Type II error when $\mu = -1/2$. Construct such a test, and give its significance level.

6. Assume that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed random variables from the Pareto distribution, which has density function

$$f(y|\theta) = (\theta - 1)y^{-\theta}$$

for $0 \leq y < \infty$, and $2 \leq \theta < \infty$.

(a) Verify that the maximum likelihood estimator of θ is

$$\hat{\theta}_{\text{MLE}} = 1 + \frac{n}{\sum_{i=1}^{n} \log Y_i}.$$

(b) Explain why $\hat{\theta}_{\text{MLE}}$ is a sufficient statistic for the estimation of θ.

(c) Verify that the Fisher Information in a single observation is

$$I(\theta) = \frac{1}{(\theta - 1)^2}.$$

(d) Say $n = 25$, and we observe a sample for which

$$\sum_{i=1}^{25} \log y_i = 5.$$

Based on this data, and on the Fisher information and the MLE, is it possible to reject the hypothesis $H_0 : \theta = 8$ in favour of $H_A : \theta \neq 8$ at the $\alpha = 0.10$ significance level?