Statistics 252 "Practice Exam" – Winter 2006

Note that all of these problems appeared on previous exams, and are relevant for this semester's course. However, completing this "practice exam" should constitute only a portion of your study and preparation for the midterm.

1. Consider a random variable Y with density function

$$f(y|\theta) = \theta^2 e^{-\theta^2 y},$$

where $y > 0$, and $\theta > 0$. Assume that $0 < \alpha_1 < \frac{1}{2}$ and $0 < \alpha_2 < \frac{1}{2}$. Using the pivotal quantity $U = \theta^2 Y$, verify that

$$\left(\sqrt{\frac{-\log(1-\alpha_1)}{Y}}, \sqrt{\frac{-\log(\alpha_2)}{Y}} \right)$$

is a confidence interval for θ with coverage probability $1 - (\alpha_1 + \alpha_2)$.

2. Suppose that the random variables $Y_1, Y_2, \ldots, Y_{10}$ are independent and identically distributed Uniform$(0, \theta)$ random variables. That is, each Y_i has density

$$f(y|\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le y \le \theta, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ. Show that $\hat{\theta}_{\text{MOM}}$ is an unbiased estimator of θ.

(b) Suppose that $Y_{(10)} = \max(Y_1, \ldots, Y_{10})$. Find an unbiased estimator which is a multiple of $Y_{(10)}$. Call it $\hat{\theta}_B$.

(c) Find the efficiency of $\hat{\theta}_{\text{MOM}}$ relative to $\hat{\theta}_B$. Which estimator do you prefer? Why?

3. Each of the following require short written answers. Very clear and brief solutions are required for full credit.

(a) Describe how to interpret a 93% confidence interval.

(b) Why do you think it is desirable to find a *minimum variance unbiased estimator*?

(continued)

4. A continuous random variable Y is said to have the Rayleigh(θ) distribution if the probability density function of Y is

$$f(y|\theta) = \frac{y}{\theta^2} \exp\left(-\frac{y^2}{2\theta^2}\right).$$

where $y > 0$ and $\theta > 0$. It turns out that

$$\mathbb{E}(Y) = \sqrt{\frac{\pi}{2}}\,\theta$$

and

$$\mathbb{E}(Y^2) = 2\theta^2.$$

(a) Determine the Fisher information $I(\theta)$ for the Rayleigh(θ) distribution.

Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed Rayleigh(θ) random variables.

(b) Compute $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ.

(c) Compute the variance of $\hat{\theta}_{\text{MOM}}$.

5. Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed random variables, each having density function

$$f(y|\theta) = \frac{\theta^{-252}}{251!}\, y^{251}\, e^{-y/\theta}$$

where $y > 0$, and $\theta > 0$ is a parameter. It is known that if $Y \sim f(y|\theta)$, then

$$\mathbb{E}(Y) = 252\,\theta \quad \text{and} \quad \text{Var}(Y) = 252\,\theta^2.$$

Let $\hat{\theta} = \overline{Y} = \dfrac{1}{n}\sum_{i=1}^{n} Y_i$.

(a) Find a multiple of $\hat{\theta}$ which is an unbiased estimator of θ. Call it $\hat{\theta}_A$

(b) Compute the Fisher information in a single observation from this density.

(c) Carefully explain why $\hat{\theta}_A$ must be the minimum variance unbiased estimator (MVUE) of θ.