

Stat 252 Winter 2006

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Example: We say that a random variable Y has the Pareto distribution if its density function is

$$f(y|\theta) = (\theta - 1)y^{-\theta}, \quad 1 \leq y < \infty$$

where θ is a parameter (satisfying $2 < \theta < \infty$). If Y_1, Y_2, \dots, Y_n are independent and identically distributed Pareto random variables, find $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ .

Solution: To find the method of moments estimator, we equate $E(Y) = \bar{Y}$ and solve for θ . We calculate

$$E(Y) = \int_1^{\infty} (\theta - 1)y^{1-\theta} dy = \left(\frac{\theta - 1}{2 - \theta} \right) y^{2-\theta} \Big|_1^{\infty} = 0 - \frac{\theta - 1}{2 - \theta} = \frac{1 - \theta}{2 - \theta}.$$

(Note that if $\theta > 2$, then $y^{2-\theta} \rightarrow 0$ as $y \rightarrow \infty$.) Thus, we find that

$$\frac{1 - \theta}{2 - \theta} = \bar{Y}$$

which upon solving for θ implies

$$\hat{\theta}_{\text{MOM}} = \frac{2\bar{Y} - 1}{\bar{Y} - 1}.$$