

(8.36) (a) If $Z \sim \mathcal{N}(0, 1)$, then using Table 4 gives

$$P(-1.96 \leq Z \leq 1.96) = 0.95.$$

That is, the normal distribution with mean 0 and variance 1 is a *parameter-free distribution*. Thus, if $Y \sim \mathcal{N}(\mu, 1)$, then

$$\frac{Y - \mu}{1} \sim \mathcal{N}(0, 1).$$

Substituting for Z gives

$$P(-1.96 \leq Y - \mu \leq 1.96) = 0.95$$

so that solving for μ in the probability statement gives

$$P(Y - 1.96 \leq \mu \leq Y + 1.96) = 0.95.$$

In other words, a 95% confidence interval for μ is

$$(Y - 1.96, Y + 1.96).$$

(b) To find a 95% upper confidence limit for a normal distribution means to find z_α such that if $Z \sim \mathcal{N}(0, 1)$, then

$$P(Z \leq z_\alpha) = 0.95.$$

Using Table 4, we find that $z_\alpha = 1.645$. Similar to **(a)**, we find that

$$P(Y - \mu \leq 1.645) = 0.95$$

so that solving for μ in the probability statement gives

$$P(\mu \geq Y - 1.645) = 0.95.$$

In other words, $Y - 1.645$ is a 95% *lower* confidence limit for μ . You should notice that because $Y - \mu \sim \mathcal{N}(0, 1)$, the inequality switched.

(c) To find a 95% lower confidence limit for a normal distribution means to find z_α such that if $Z \sim \mathcal{N}(0, 1)$, then

$$P(Z \geq z_\alpha) = 0.95.$$

Again using Table 4, we find that $z_\alpha = -1.645$. Similar to **(a)**, we find that

$$P(Y - \mu \geq -1.645) = 0.95$$

so that solving for μ in the probability statement gives

$$P(\mu \leq Y + 1.645) = 0.95.$$

In other words, $Y + 1.645$ is a 95% *upper* confidence limit for μ . You should notice that because $Y - \mu \sim \mathcal{N}(0, 1)$, the inequality switched.

(Remark: Technically, the answers to (b) and (c) should be switched, but because I am most concerned that you intuitively understand what is going on, that is a minor concern.)

(8.37) (a) If $Z \sim \chi_1^2$, then using Table 6 gives

$$P(0.0009821 \leq Z \leq 5.02389) = 0.95.$$

Since the pivotal quantity Y^2/σ^2 has a χ_1^2 distribution, substituting in for Z in the probability statement gives

$$P\left(0.0009821 \leq \frac{Y^2}{\sigma^2} \leq 5.02389\right) = 0.95$$

so that

$$P\left(\frac{Y^2}{5.02389} \leq \sigma^2 \leq \frac{Y^2}{0.0009821}\right) = 0.95$$

In other words, a 95% confidence interval for σ^2 is

$$\left(\frac{Y^2}{5.02389}, \frac{Y^2}{0.0009821}\right).$$

(b) To find a 95% upper confidence limit for a chi-squared distribution with $df = 1$ means to find χ_α such that if $Z \sim \chi_1^2$, then

$$P(Z \leq \chi_\alpha) = 0.95.$$

Using Table 6 gives $\chi_\alpha = 3.84146$ so that

$$P(Z \leq 3.84146) = 0.95.$$

Substituting in for Z and solving for σ^2 in the probability statement gives

$$P\left(\sigma^2 \geq \frac{Y^2}{3.84146}\right) = 0.95.$$

In other words, $Y^2/3.84146$ is a 95% lower confidence limit for σ^2 . You should notice that because $Y^2/\sigma^2 \sim \chi_1^2$, the inequality switched.

(c) To find a 95% lower confidence limit for a chi-squared distribution with $df = 1$ means to find χ_α such that if $Z \sim \chi_1^2$, then

$$P(Z \geq \chi_\alpha) = 0.95.$$

Using Table 6 gives $\chi_\alpha = 0.0039321$ so that

$$P(Z \geq 0.0039321) = 0.95.$$

Substituting in for Z and solving for σ^2 in the probability statement gives

$$P\left(\sigma^2 \leq \frac{Y^2}{0.0039321}\right) = 0.95.$$

In other words, $Y^2/0.0039321$ is a 95% upper confidence limit for σ^2 . You should notice that because $Y^2/\sigma^2 \sim \chi_1^2$, the inequality switched.

(Remark: Technically, the answers to (b) and (c) should be switched, but because I am most concerned that you intuitively understand what is going on, that is a minor concern.)

(8.38) (a) From (8.37), we find that

$$P\left(\frac{Y^2}{5.02389} \leq \sigma^2 \leq \frac{Y^2}{0.0009821}\right) = 0.95.$$

Since the square root function is monotonic, we can conclude

$$P\left(\sqrt{\frac{Y^2}{5.02389}} \leq \sqrt{\sigma^2} \leq \sqrt{\frac{Y^2}{0.0009821}}\right) = 0.95.$$

Since Y is a *non-negative* random variable, and since $\sigma > 0$, we conclude

$$P\left(\frac{Y}{\sqrt{5.02389}} \leq \sigma \leq \frac{Y}{\sqrt{0.0009821}}\right) = 0.95,$$

or, in other words, a 95% confidence interval for σ is

$$\left(\frac{Y}{\sqrt{5.02389}}, \frac{Y}{\sqrt{0.0009821}}\right).$$

(b) Similarly, $Y/\sqrt{0.0039321}$ is a 95% upper confidence limit for σ .

(c) Similarly, $Y/\sqrt{3.84146}$ is a 95% lower confidence limit for σ .

(8.43) From the data presented, we find that $n = 2374$ adults in the continental US were interviewed, of which 1912 were registered voters. Thus, if p denotes the true proportion of registered voters in the continental US, then from this we conclude

$$\hat{p} = \frac{1912}{2374}.$$

Thus, an approximate 99% confidence interval for p is given by

$$\hat{p} \pm z_a \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{or} \quad \frac{1912}{2374} \pm 2.575 \cdot \sqrt{\frac{1912/2374 \cdot 462/2374}{2374}}.$$

In other words, at the 99% confidence level, the proportion of adults in the continental US registered to vote is between 0.784 and 0.826.

(8.25) (a) Let p_1 denote the proportion of Americans who ate the recommended amount of fibrous foods in 1983, and let p_2 denote the proportion who ate the recommended amount in 1992. The data then yield $\hat{p}_1 = 0.59$ and $\hat{p}_2 = 0.53$. The estimated standard errors are easily calculated as:

$$\hat{\sigma}_{\hat{p}_1} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n}} = \sqrt{\frac{0.59 \cdot 0.41}{1250}} \quad \text{and} \quad \hat{\sigma}_{\hat{p}_2} = \sqrt{\frac{\hat{p}_2(1-\hat{p}_2)}{n}} = \sqrt{\frac{0.53 \cdot 0.47}{1251}}.$$

Thus, a point estimate for the difference is given by $\hat{p}_1 - \hat{p}_2 = 0.59 - 0.53 = 0.06$. This indicates that there was a 6% *decrease* in the proportion of Americans who were eating the recommended amount of fibrous foods in 1993 compared with 1982. A bound on the error of estimation is

$$2\sqrt{\hat{\sigma}_{\hat{p}_1}^2 + \hat{\sigma}_{\hat{p}_2}^2} \approx 0.04.$$

(b) Note that the answer in (a) yields an approximate 95% confidence interval of

$$0.06 \pm 0.04 = (0.02, 0.10).$$

Since this interval *does not cover* 0, there is statistically significant evidence to indicate that there has been a demonstrable decrease in the proportion of Americans who ate the recommended amount of fibrous foods in 1993 compared with 1982.

(8.50) (a) This problem is similar to (8.25). Using the answers to (8.25), and the fact that the critical value for a 98% normal confidence interval is 2.33, we conclude that an approximate 98% confidence interval for the difference is

$$\hat{p}_1 - \hat{p}_2 \pm 2.33\sqrt{\hat{\sigma}_{\hat{p}_1}^2 + \hat{\sigma}_{\hat{p}_2}^2} \quad \text{or} \quad 0.06 \pm 0.046.$$

(b) As before, since this interval *does not cover* 0, there is statistically significant evidence to indicate that there has been a demonstrable decrease in the proportion of Americans who ate the recommended amount of fibrous foods in 1993 compared with 1982.