Statistics 252 "Practice" Midterm #2– Winter 2005

This exam has 5 problems and is worth 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, as well as an $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes. A dictionary will be provided. You may also consult Table 4: Normal Curve Areas.*

Name: ____________________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	
5	

TOTAL: ____________

Note that all of these problems appeared on previous exams, and are relevant for this semester's course. However, completing this "practice" exam should constitute only a portion of your study and preparation for the midterm.

1. (*12 points*) Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables. It is known that $\sigma^2 = 4$, but μ is unknown. You want to test $H_0 : \mu = 0$ against $H_A : \mu > 0$ by rejecting H_0 when $\overline{Y} > 3.92/\sqrt{n}$.

(a) Verify that this test has significance level $\alpha = 0.025$.

(b) Say you are planning an experiment for which the data will be analyzed by this hypothesis test. How large a sample should you collect if you would like the test to have power 0.9 when $\mu = 0.5$?

(*NOTE: Think through this problem. Do not look up formulæ we did not cover in class.*)

2. (*8 points*) Let $Y_1, \ldots, Y_{100}$ be independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables. It is known that $\sigma^2 = 25$, but μ is unknown. Consider testing $H_0 : \mu = 0$ against $H_A : \mu < 0$ by rejecting H_0 for small values of the sample mean $\overline{Y}$. The user of the test would like to treat the hypotheses somewhat symmetrically. In particular, she wants the probability of Type I error to be equal to the probability of Type II error when $\mu = -1/2$. Construct such a test, and give its significance level.

3. (*12 points*) Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed with density function

$$f(y|\theta) = \theta \exp(-\theta y)$$

where $y > 0$ and $\theta > 0$. As usual, let $\overline{Y}$ denote the sample mean given by

$$\overline{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i.$$

It is known that the maximum likelihood estimator of θ is $\hat{\theta}_{\mathrm{MLE}} = 1 / \overline{Y}$.

(a) Consider testing $H_0 : \theta = \theta_0$ against $H_A : \theta \neq \theta_0$. Verify that the rejection region for the generalized likelihood ratio test of these hypotheses is of the form

$$\{\overline{Y} \exp(-\theta_0 \overline{Y}) \leq C\}$$

for some suitable constant C.

(b) Suppose, to be specific, that $\theta_0 = 1$, and that a random sample of size $n = 10$ is conducted. If the observed data yield $\overline{Y} = 1.25$, perform the generalized likelihood ratio test at the approximate significance level $\alpha = 0.10$.

(*Hint: Use the limiting result about a functional of the likelihood ratio Λ.*)

4. (*8 points*)

(a) In the context of Stat 252, clearly define what is meant by a *significance level* α *hypothesis test.*

(b) In the context of simple linear regression as studied in Stat 252, discuss the difference(s) between a *confidence interval* and a *prediction interval.*

5. (*10 points*) Suppose that $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are independent and identically distributed $\mathcal{N}(0, \sigma^2)$ random variables. Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed random variables, with

$$Y_i = \beta x_i + \varepsilon_i$$

for each i. Under these assumptions, it is known that Y_i is also normally distributed. That is,

$$Y_i \sim \mathcal{N}(\beta x_i, \sigma^2).$$

Find the maximum likelihood estimator of β.

(*Hint: Find the joint density function of the* Y_i *and thus the likelihood function.*)

Bonus: Show (with minimal computations by virtue of your computation above) that $\hat{\beta}_{\mathrm{MLE}}$ is also the least squares estimator of β. (Compare to Exercise 11.6.)