Stat 252.01 Winter 2005
Assignment #7

This assignment is due at the beginning of class on **Monday, March 14, 2005**. You must submit all problems. You are encouraged to form study groups and collaborate with others on this assignment. However, the final work you submit must be your own. A piece of advice: *the assignments are worth very little in the computation of your final grade. It is better to suffer through not understanding something now, rather than copying from a friend just for the sake of completion. You will not have that luxury on the exams.* YOUR ASSIGNMENT MUST BE STAPLED AND PROBLEM NUMBERS CLEARLY LABELLED. UNSTAPLED ASSIGNMENTS WILL NOT BE ACCEPTED! DO NOT CROWD YOUR WORK. DO NOT WRITE IN MULTIPLE COLUMNS. *Note that if there are answers in the back of the book, then you need to be especially certain to explain your answer.*

Throughout this assignment, the "bar" notation for the mean has its usual meaning:

$$\overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i.$$

1. Consider the usual linear regression model $Y = \beta_0 + \beta_1 x + \varepsilon$ where the random variable ε has mean 0. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the usual least squares estimates (as given in the shaded box on page 539 of our textbook). In other words, $\mathbb{E}(Y) = \beta_0 + \beta_1 x$ and the estimated trend line is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$. As usual, let $y_1, \ldots, y_n$ be observations of the random variable Y, and let the ith error (or residual) be denoted $y_i - \hat{y}_i$.

- Show that $\sum_{i=1}^{n}(y_i - \hat{y}_i) = 0$. (This is why we consider the *squared* residuals when doing least squares.)
- Show that the estimated trend line passes through the point $(\overline{x}, \overline{y})$.

2. Do the following exercises from Wackerly, et al.

- #11.1, page 540 (Be neat and careful, and carry out all of your calculations by hand.)
- #11.6, page 542

3. Suppose that $\varepsilon_1, \ldots, \varepsilon_n$ are independent and identically distributed random variables, each with mean 0 and variance σ^2. Suppose further that the random variables $Y_1, \ldots, Y_n$ are defined by

$$Y_i = \mu + \varepsilon_i$$

for $i = 1, \ldots, n$. Prove that $\overline{Y}$ is the least squares estimate of μ.