Stat 252.01 Winter 2005
Assignment #5

This assignment is due at the beginning of class on **Monday, February 28, 2005**. You must submit all problems. You are encouraged to form study groups and collaborate with others on this assignment. However, the final work you submit must be your own. A piece of advice: *the assignments are worth very little in the computation of your final grade. It is better to suffer through not understanding something now, rather than copying from a friend just for the sake of completion. You will not have that luxury on the exams.* YOUR ASSIGNMENT MUST BE STAPLED AND PROBLEM NUMBERS CLEARLY LABELLED. UNSTAPLED ASSIGNMENTS WILL NOT BE ACCEPTED! DO NOT CROWD YOUR WORK. DO NOT WRITE IN MULTIPLE COLUMNS. *Note that if there are answers in the back of the book, then you need to be especially certain to explain your answer.*

1. Suppose that $Y_1, \ldots, Y_n$ are iid from the $\mathcal{N}(\mu, \sigma^2)$ distribution, where σ^2 is known, but μ is unknown. Consider testing $H_0 : \mu = \mu_0$ against $H_A : \mu > \mu_0$ by rejecting H_0 if $Z > 1.65$, where

$$Z = \frac{\overline{Y} - \mu_0}{\sigma/\sqrt{n}}.$$

In class we showed that this test has significance level 0.05.

(a) Assume that $\mu_0 = 0$, $\sigma^2 = 25$, and $n = 4$. What is the power of the test when $\mu = 1$, when $\mu = 2$, and when $\mu = 3$?

(b) Repeat part **(a)** assuming a larger sample size of $n = 16$. Do you have any comments about the comparison of power for the two sample sizes?

2. Suppose that $X_1, \ldots, X_{10}$ are iid from the $\mathcal{N}(\mu, \sigma^2)$ distribution, where both μ and σ^2 are unknown. As usual, let S^2 denote the sample variance. Consider testing $H_0 : \sigma^2 = 1$ against $H_A : \sigma^2 > 1$, by rejecting H_0 when $S^2 > c$.

(a) Determine c so that this test has significance level 0.1.

(b) What is the power of this test when $\sigma^2 = 2$ and when $\sigma^2 = 3$?

NOTE: Table 6 in the back of the textbook is not complete enough for these calculations, so I have provided some quantiles of the χ^2_9 distribution below. You can give your answers to the accuracy permitted by this information.

α	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
χ^2_α	3.33	4.17	4.82	5.38	5.90	6.39	6.88	7.36	7.84	8.34	8.86	9.41	10.01	10.66

α	0.25	0.20	0.15	0.10	0.05
χ^2_α	11.39	12.24	13.29	14.68	16.92

(continued)

3. Consider observing a single random variable X from an Exponential(λ) distribution. We want to test $H_0 : \lambda = 1$ against $H_A : \lambda = 1/2$ by rejecting H_0 is $X < c$. (For the exponential distribution, smaller values of the parameter λ tend to produce smaller values of X.) By changing c, we will change both α and β, the probabilities of a Type I and Type II error, respectively. Can you find a direct relationship between α and β which illustrates the tradeoff between them?

4. Suppose that $X_1, \ldots, X_n$ are iid from the Exponential(λ) distribution. Starting with an approximate confidence interval for λ based on the Fisher information, construct a test of $H_0 : \lambda = 1/10$ against $H_A : \lambda \neq 1/10$ at (approximate) significance level 0.1.

5. Do the following exercises from Wackerly, et al.

- #10.10, page 474
- #10.19, page 476
- #10.38, page 482