Math/Stat 251 Midterm #1 – October 21, 2015

This exam has 5 problems on 4 numbered pages and is worth a total of 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements. For questions with multiple parts, all parts are equally weighted.*

This exam is closed-book, although one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is allowed. No other aids are allowed.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ____________________________________ Instructor: Michael Kozdron

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TOTAL: ____________

1. (*10 points*) Health Canada is responsible for protecting the safety of Canada's food supply. As such, Health Canada carries out tests of food products to determine whether or not they comply with existing regulations and standards.

A producer of prepackaged lunch snacks states that harmful bacterial is found in 3 out of every 1000 snacks. Suppose that Health Canada selects 5 of this producer's prepackaged lunch snacks at random. Assume that each snack is selected independently of the others.

(a) What is (an expression for) the probability that none of the 5 prepackaged lunch snacks contains harmful bacteria? You do not need to simply your expression.

(b) What is (an expression for) the probability that at least 3 of the 5 prepackaged lunch snacks contain harmful bacteria? You do not need to simply your expression.

2. (*8 points*)

(a) Suppose that A and B are *disjoint* events satisfying $\mathbf{P}(A^c) = 0.4$ and $\mathbf{P}(B^c) = 0.9$. Determine $\mathbf{P}(A \cup B)$.

(b) Suppose that A and B are *independent* events with $\mathbf{P}(A \cap B) = 0.3$ and $\mathbf{P}(A) = 0.6$. Determine $\mathbf{P}(A \cup B)$.

3. (*5 points*) Suppose that A and B are independent events. Show that A^c and B^c are also independent events.

4. (*15 points*) Andrew, an absent minded professor, owns 14 socks. The socks are identical in every aspect except for their colour: 2 of them are white, 4 of them are blue, and 8 of them are red. Andrew selects the socks he will wear by choosing two of them uniformly at random from among all his socks. Express your answers to the following questions as a fraction in lowest terms.

(a) What is the probability that he selects two blue socks?

(b) What is the probability that he selects two socks of the same colour?

(c) Suppose that Andrew selects two socks of the same colour. What is the probability that both those socks are blue?

5. (*12 points*) Suppose that

$$F(x) = \begin{cases} 0, & x < 0, \\ cx^2, & 0 \le x \le 2, \\ 1, & x > 2. \end{cases}$$

(a) Determine the unique value of c such that F is a legitimate distribution function.

For the remaining parts of this problem, suppose that X is a random variable having distribution function F using the value of c you found in **(a)**.

(b) Determine $f(x)$, the density function of X.

(c) Determine $\mathbf{P}(1 < X \le 2)$.