Stat 251 Midterm Exam 1
Date: October 15, 2014
Total 50 marks

- **You may ask me about interpretations of these questions!**
- Please write your complete solutions in the booklets provided. Showing all of your work is important!

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1. A balanced die is tossed six times, and the uppermost face is recorded each time.
   (a) (3 marks) Carefully describe the sample space and give the size of the sample space.
   (b) (4 marks) What is the probability that the numbers recorded are 1,2,3,4,5, and 6, in any order?
   (c) (3 marks) What is the probability that no 5 appears on all six tosses, given that no 5 has appeared on the first 3 tosses?

2. Suppose A and B are two given events.
   (a) (3 marks) Carefully define the term '*the events A and B are independent*'.
   (b) (2 marks) Is the following state true or false? Explain your reasoning. "If the events A and B are mutually exclusive, then these events must be independent."
   (c) (5 marks) Prove that $P(A \cap B) \geq 1 - P(A^c) - P(B^c)$ (Bonferroni inequality).

3. Certain medical case histories indicate that different illnesses may produce identical symptoms. Suppose that a certain set of symptoms, denoted H, occur only when any one of three illnesses I_1, I_2, or I_3, occurs. Assuming that the simultaneous occurrence of more than one of these illnesses is impossible, suppose $P(I_1) = .01, P(I_2) = .005, P(I_3) = .02$. The probabilities of developing symptoms H given each illness is: $P(H|I_1) = .9, P(H|I_2) = .95, P(H|I_3) = .75$.
   (a) (2 marks) Carefully state Bayes' Theorem for a given partition.
   (b) (4 marks) What is probability that a random person develops symptoms H?
   (c) (4 marks) Assuming that an ill person exhibits symptoms H, what is the probability that the person has illness I_1?

4. A potential customer for an \$85,000 fire insurance policy has a home that may sustain a total loss in a given year with probability 0.001, and a 50% loss with probability 0.01.
   (a) (3 marks) Define the term the '*expected value of a random variable*'.
   (b) (3 marks) If X is defined as the profit for this insurance company and C is the premium to be charged, construct a probability distribution for X.
   (c) (4 marks) If X is defined as above, then find the premium that should be charged so the insurance company can expect to earn \$100 for each such policy.

5. Ten percent of the engines manufactured on a assembly line are defective. If engines are randomly selected one at a time and tested:
   (a) (2 marks) What is the probability that the first nondefective engine will be found on the second trial?
   (b) (3 marks) What is the probability that the third nondefective engine will be found on or before the fifth trial?
   (c) (2 marks) Find the mean and variance for the number of trials on which the third nondefective engine is found.
   (d) (3 marks) Given that the first two engines tested were defective, what is the probability that at least two more engines must be tested before the first nondefective engine is found?