Math/Stat 251 Fall 2015
September 9, 2015
Thinking About Probability

Historically, probability arose as a way to describe games of chance. By the 18th century, considerably general methods for calculating odds in games of chance had been invented. During this period, the discipline of statistics was also being developed. Statistical data was being compiled for both mortality rates and political economy. Merchants interested in protecting their investments against shipwrecks led to the growth of insurance companies and the actuarial profession. It was soon realized that probabilistic models could help study these problems. In the late 19th and early 20th centuries, physicists discovered that energy states of atoms were described only by probability distributions. As quantum mechanics evolved, it became intricately linked with probability. By the 1940s, probability was established as a rigorous mathematical discipline, and many of the early gambling paradoxes that stumped 18th century thinkers had been resolved. Nowadays, probability is a mathematical discipline in its own right, and some people actually study probability for the sake of probability. However, probability is extremely important because of the vastness and variety of its applications. More continue to be discovered, but as a small sample, these applications include population genetics, data mining, machine learning, image processing, wireless communication system design, web search engine design, financial engineering, stock option pricing, volatility analysis, insurance risk, and polymer chemistry. Furthermore, probability is the language of statistics and the foundations of statistical inference are based entirely on theoretical probability. In fact, if you end up taking a course like STAT 252, you will start to explore this probabilistic basis for statistical inference.

In order to succeed in a course like MATH/STAT 251, it is important that you develop your intuition for probability as well as your ability to manipulate mathematical objects (functions, derivatives, integrals, etc.). The whole point is that probability started because people wanted to make money gambling. It then took mathematics to clean up probability and make it a respectable subject!

Please read through the following problems and attempt to solve them intuitively. We will be discussing these problems in class on **September 11, 2015**, so you do not need to prepare a formal write-up to submit. However, you will derive the most benefit if you make a concerted effort to solve them before that class.

1. Each year, the Canadian coffee and donut chain Tim Hortons brings back its *Roll Up The Rim To Win* contest. The basic idea is that each coffee cup serves as a game piece and by looking under the rim of your cup, you can determine whether or not you've won a prize. The official rules state that there is a *1-in-9 chance of winning a prize.*

(a) Interpret this last phrase. In other words, what does Tim Hortons mean when they claim there is a 1-in-9 chance of winning a prize?

(b) Suppose that you buy 8 cups of coffee and do not win a prize with any of those cups. Are you *guaranteed* to win a prize when you buy your 9th cup of coffee?

(c) Suppose that you buy 18 cups of coffee. How many prizes will you win? How many prizes do you *expect* to win?

2. The Lotto 6/49 is a Canadian national lottery in which 49 balls numbered 1 through 49 are placed into a large drum. Six balls are then successively drawn out at random. In order to play the lottery, a person selects 6 numbers and buys a ticket. If those 6 numbers picked match the 6 numbers drawn (order is unimportant), then the player wins the jackpot.

(a) Determine the probability that a player wins the jackpot.

(b) There is a \$10 prize given if a player matches any 3 numbers. Determine the probability that a player wins \$10.

3. A very simple model for the price of a stock is the following. Each day the value of the stock is equally likely to either increase by \$1 or to decrease by \$1. Suppose that today (time $t = 0$) the value of the stock is \$13.

(a) What is the probability that on day 3 (time $t = 3$), the value of the stock is \$16? (Note that this can only happen if the stock goes up, up, up.)

(b) What is the probability that on day 3, the value of the stock is \$12? (Note that there are several ways for this to happen. You will need to enumerate them.)

4. Another simple model for the price of a stock is the following. Each day there is a 60% chance that the value of the stock increases by \$1, while there is a 40% chance that the value of the stock decreases by \$1. What is the probability that on day 3, the value of the stock is \$12? (The solution is different than the one to the previous problem.) *These models of stock price movement are known as random walks and are the basis for more sophisticated models that arise in finance such as geometric Brownian motion.*

5. Texas Hold'em is a widely popular variant of the card game poker. The basic idea is that each player is dealt 2 cards face down (that is, they do not show these cards to the other players) and there are 3 community cards dealt face up in the centre of the table. The player with the best *poker hand* based on these 5 cards wins. While there is no skill whatsoever involved at forming the poker hand (since it is determined by the 5 cards dealt), the intrigue and psychology revolve around the betting stages while the community cards are revealed and players try to outwit their opponents and force them to fold or continue with a weak hand. Probability can be used to determine the likelihood of a particular poker hand. More difficult calculations can be done to determine the probability of a particular poker hand assuming the opponent is holding certain cards, say a pair of aces. Here is one example. Suppose that 5 cards are dealt at random from a standard 52 card deck. What is the probability of having exactly 3 aces and 2 kings?