

Math/Stat 251 Fall 2015

Tips for Using Bayes' Rule (September 28, 2015)

In order to use Bayes' Rule correctly it is absolutely vital that you VERY CAREFULLY define appropriate events. In the statement of the problem, you will be given a conditional probability for one event given another event, and you will want to determine a conditional probability for those events swapped. We will call these events A and B_1 so that you are given the conditional probability

$$\mathbf{P}(A | B_1).$$

You want to determine

$$\mathbf{P}(B_1 | A).$$

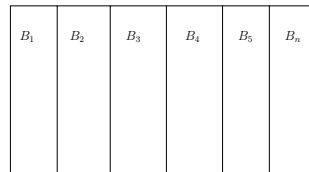
Write down the basic version of Bayes' Rule using the fact that $A \cap B_1 = B_1 \cap A$; that is, $\mathbf{P}(A \cap B_1) = \mathbf{P}(B_1 \cap A)$ and so

$$\mathbf{P}(A | B_1) \mathbf{P}(B_1) = \mathbf{P}(B_1 | A) \mathbf{P}(A)$$

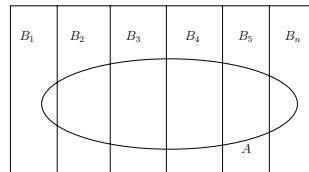
implying that

$$\mathbf{P}(B_1 | A) = \frac{\mathbf{P}(A | B_1) \mathbf{P}(B_1)}{\mathbf{P}(A)}. \quad (*)$$

Hence, we need to determine the values of the three probabilities on the right-side of the previous equation (*). As noted already, in the statement of the problem you will be given $\mathbf{P}(A | B_1)$, or at least enough information to determine it such as $\mathbf{P}(A^c | B_1)$. You will also be given $\mathbf{P}(B_1)$, or at least enough information to determine it such as $\mathbf{P}(B_1^c)$. The key step will often be to use the law of total probability to determine $\mathbf{P}(A)$. Here is the key. You want to *partition* the sample space. You will need to use B_1 for this. The event B_1 will typically give clues to how the other events B_2, \dots, B_n should be defined. Schematically, you have the following.



Now consider the event A .



Now, write A as a disjoint union of the ways it can intersect the partition B_1, B_2, \dots, B_n , namely

$$\begin{aligned} \mathbf{P}(A) &= \mathbf{P}(A \cap B_1) + \mathbf{P}(A \cap B_2) + \dots + \mathbf{P}(A \cap B_n) \\ &= \mathbf{P}(A | B_1) \mathbf{P}(B_1) + \mathbf{P}(A | B_2) \mathbf{P}(B_2) + \dots + \mathbf{P}(A | B_n) \mathbf{P}(B_n). \end{aligned}$$

This is the Law of Total Probability. Notice that this expression includes the same term as the numerator, namely $\mathbf{P}(A | B_1) \mathbf{P}(B_1)$, but also includes more terms. Information about these other terms will also be given in the statement of the problem.

Remark. Problem #3 on Assignment #3 (the one about Elvis having a twin brother) can be solved by following this template precisely.

Remark. Sometimes a tree diagram is useful for figuring out the possible events to use for the partition B_1, \dots, B_n .