Math/Stat 251 Fall 2015
Practice Problems for Midterm #2 (November 16, 2015)

Problem 1. Suppose that X is a continuous random variable with density function

$$f(x) = \begin{cases} \frac{3}{7}x^2, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Verify that f is, in fact, a legitimate density function.

(b) Compute $\mathbb{E}(X)$, the expected value (or mean or average) of X.

(c) Compute $\text{Var}(X)$, the variance of X.

(d) Determine $F(x)$, the distribution function of X.

(e) Determine the *median* of X.

Problem 2. Suppose that X_1 and X_2 are independent continuous random variables each having common *distribution* function

$$F(x) = \begin{cases} 1 - xe^{-x} - e^{-x}, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

(a) Determine $f(x)$, their common density function.

(b) Suppose that $Y_1 = \min\{X_1, X_2\}$. Determine $f_{Y_1}(y)$, the density function of Y_1.

(c) Suppose that $Y_2 = \max\{X_1, X_2\}$. Determine $f_{Y_2}(y)$, the density function of Y_2.

(d) Let $Z_1 = Y_1^3$. Determine $f_{Z_1}(z)$, the density function of Z_1.

(e) Let $Z_2 = \sqrt{Y_2}$. Determine $f_{Z_2}(z)$, the density function of Z_2.

Problem 3. Suppose that X and Y are independent, continuous random variables. If the density function of X is $f_X(x) = xe^{-x}$ for $x \ge 0$, and the density function of Y is $f_Y(y) = e^{-y}$ for $y \ge 0$, use the law of total probability to determine $\mathbf{P}(X < Y)$. *Hint*: It is probably easier to condition on the value of X.

Problem 4. Suppose that X is a continuous random variable with distribution function $F(x)$ and density function $f(x)$. Suppose further that f is continuous. Use the law of the unconscious statistician to show that $\mathbb{E}[F(X)] = 1/2$.