UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
Statistics 251-001
Final Examination
2014-30

Time: 3 hours
Instructor: Dr. S. Fallat
Total Marks: 100

Show all of your work, making sure to explain all necessary steps. Use the booklets provided to answer the questions below. Scrap paper is available for rough work. You are allowed one formula sheet, but no other notes or books are permitted! Calculators are OK.

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1. Given the moment generating function $m(t) = e^{3t+8t^2}$ for a random variable Y:
   (a) (3 marks) Describe the random variable Y.
   (b) (5 marks) Find the moment generating function for the random variable $Z = \frac{1}{4}(Y - 3)$.
   (c) (4 marks) Find $E(Z)$.

2. A committee of three people is to be randomly selected from a group containing four Republicans, three Democrats, and two Independents. Let Y_1 and Y_2 denote numbers of Republicans and Democrats, respectively, on this committee.
   (a) (4 marks) What is the joint probability distribution of Y_1 and Y_2?
   (b) (4 marks) What is the marginal distribution of Y_1. What kind of random variable is Y_1?
   (c) (3 marks) Find $P(Y_1 = 1|Y_2 \geq 1)$.
   (d) (5 marks) Find the conditional probability function $p(y_1|Y_2 = 1)$.
   (e) (3 marks) Find $\text{Cov}(Y_1, Y_2)$.
   (f) (2 marks) Are Y_1 and Y_2 independent? Explain!

3. A test for a disease correctly detects the disease in 90% of the patients that actually have the disease. Also, if a person does not have the disease the test correctly reports no disease with probability .9. Suppose 1% of the population has the disease.

   (a) (3 marks) State carefully Bayes' Theorem for a given partition.

   (b) (4 marks) What is the probability that the test detects the disease in a random patient?

   (c) (5 marks) If the test detects the disease in a random patient, what is the probability that the patient has the disease?

4. Suppose customers arrive at a checkout counter according to a Poisson distribution at an average rate of seven per hour.

   (a) (2 marks) Carefully describe a general Poisson random variable with parameter λ.

   (b) (3 marks) What is the probability exactly three customers arrive in a given two hour period?

   (c) (4 marks) What is the probability that a total of three customers arrive between 1:00-2:00 pm and 3:00-4:00 pm?

   (d) (5 marks) Prove that the moment generating function for a Poisson random variable with parameter λ is $m(t) = e^{\lambda(e^t - 1)}$.

5. Let Y_1 and Y_2 be two random variables that satisfy: $E(Y_1) = 4$, $E(Y_2) = -1$, $V(Y_1) = 2$, and $V(Y_2) = 8$.

   (a) (2 marks) Define the term "covariance of Y_1 and Y_2".

   (b) (2 marks) What is $\text{Cov}(Y_1, Y_1)$?

   (c) (4 marks) Is it possible that $\text{Cov}(Y_1, Y_2) = 7$? Carefully explain your reasoning.

   (d) (4 marks) What is possible the largest possible value of $\text{Cov}(Y_1, Y_2)$? Carefully explain your reasoning.

6. Let Y_1 and Y_2 be independent Poisson random variables with means λ_1 and λ_2, respectively.

   (a) (3 marks) Carefully define the term "X and Y are independent random variables".

   (b) (5 marks) Find the probability mass function of $Y_1 + Y_2$, and state what kind of random variable is $Y_1 + Y_2$.

   (c) (6 marks) Find the probability mass function of Y_1, given that $Y_1 + Y_2 = m$.

7. The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with distribution function given by:

$$F(y) = \begin{cases} 1 - e^{-y^2}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) (3 marks) Carefully describe the properties of a distribution function.

(b) (3 marks) Find the 30th quantile, namely c, such that 30% of the probability lies to the left of c.

(c) (4 marks) Find the density function, $f(y)$.

(d) (2 marks) Find the probability that a transistor operates for at least 200 hours.

(e) (3 marks) Find $P(Y > 100|Y \leq 200)$.