Math/Stat 251 Fall 2015
Assignment #1

This assignment is due on Wednesday, September 16, 2015. You do not need to submit the problems for grading.

Math/Stat 251 provides an introduction to the mathematical theory of probability. The tools that we will use in this class include integration-by-substitution and integration-by-parts from first year calculus. The beginning of the term is a good time to review this material. We will not be reviewing techniques of integration when we encounter integrals in a few weeks. Here are a couple of direct computations to help you refresh your skills, and to illustrate the type of integrals we will need to consider.

Single-Variable Calculus

1. Compute $\int_0^\infty e^{-2x}\,\mathrm{d}x$.

2. Compute $\int_0^\infty xe^{-2x}\,\mathrm{d}x$.

3. Compute $\int_0^\infty x^2 e^{-2x}\,\mathrm{d}x$.

4. Compute $\int_0^\infty x^3 e^{-2x}\,\mathrm{d}x$.

5. Compute $\int_0^\infty x^{-2/3} e^{-x^{1/3}}\,\mathrm{d}x$.

6. For any $a > 0$, compute $\int_0^\infty x^{1/a-1} e^{-x^{1/a}}\,\mathrm{d}x$.

7. Compute $\int_0^\infty x^{1/3} e^{-2x^{1/3}}\,\mathrm{d}x$.

8. Compute $\int_0^\infty xe^{-x^2}\,\mathrm{d}x$.

9. For any $a > 0$, compute $\int_0^\infty xe^{-ax^2}\,\mathrm{d}x$.

10. Compute $\int_0^1 x(1-x)^3\,\mathrm{d}x$.

11. Compute $\int_0^1 x^2(1-x)^3\,\mathrm{d}x$.

12. Compute $\int_{-\infty}^\infty \frac{1}{x^2+1}\,\mathrm{d}x$.

13. Compute $\int_0^\infty \frac{x}{x^2+1}\,\mathrm{d}x$.

14. Compute $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\, dx$.

15. For any $a > 0$, compute $\displaystyle\int_{a}^{\infty} \frac{1}{x^3}\, dx$.

16. For any $a > 0$ and for any $b > 1$, compute $\displaystyle\int_{a}^{\infty} \frac{1}{x^b}\, dx$.

Some Sums

1. Compute the exact value of $\displaystyle\sum_{j=0}^{\infty} 3^{-j}$. (Hint: It is a particular geometric series.)

2. Compute the exact value of $\displaystyle\sum_{j=1}^{\infty} j3^{-j}$. (You may not have seen this one before.)

3. Compute the exact value of $\displaystyle\sum_{j=0}^{\infty} \frac{3^{-j}}{j!}$. (Hint: It is a particular power series.)