Math/Stat 251 Fall 2015
Assignment #5

Solutions should be completed, but not submitted, by Wednesday, October 14, 2015.

1. For each of the following functions f, determine whether or not there is a value of c that makes f a legitimate probability density function. If such a c exists, compute its value. If such a c does not exist, carefully explain why.

(a) $f(x) = \begin{cases} cx^{-2}, & x \geq 1, \\ 0, & x < 1. \end{cases}$

(b) $f(x) = \begin{cases} cx^{-1}, & x \geq 1, \\ 0, & x < 1. \end{cases}$

(c) $f(x) = \begin{cases} cx^2e^{-x}, & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$

(d) $f(x) = \begin{cases} cx^2e^{-x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

(e) $f(x) = \begin{cases} cxe^{x}, & x \leq 0, \\ 0, & x > 0. \end{cases}$

(f) $f(x) = \begin{cases} cxe^{x}, & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$

2. In each of the following cases, compute $\mathbf{P}\{0 < X < 2\}$ where the random variable X has the given probability density function.

(a) $f(x) = \begin{cases} 7e^{-7x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

(b) $f(x) = \begin{cases} xe^{-x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

3. Suppose that the random variable X has density function

$$f(x) = \begin{cases} \frac{1}{8}x & 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the value of a such that $\mathbf{P}\{X \leq a\} = \dfrac{1}{2}$.

(b) Determine the value of a such that $\mathbf{P}\{X \geq a\} = \dfrac{1}{4}$.

4. Suppose that the random variable X has distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{8}x^3, & 0 \le x \le 2, \\ 1 & x > 2. \end{cases}$$

(a) Compute $\mathbf{P}\{X \le 1\}$.

(b) Compute $\mathbf{P}\{0.5 \le X \le 1.5\}$.

(c) Determine the value of a such that $\mathbf{P}\{X \le a\} = \dfrac{1}{2}$.

(d) Determine the value of a such that $\mathbf{P}\{X \ge a\} = \dfrac{1}{4}$.

5. Suppose that X is a normally distributed random variable with density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for $-\infty < x < \infty$. Using a table of normal probabilities, compute each of the following probabilities accurate to 4 decimal places.

(a) $\mathbf{P}\{X > 1\}$.

(b) $\mathbf{P}\{X < 1\}$.

(c) $\mathbf{P}\{X \le 1\}$.

(d) $\mathbf{P}\{-1 \le X \le 1\}$.

(e) $\mathbf{P}\{X \le 2\}$.

(f) $\mathbf{P}\{X \ge -2\}$.

(g) $\mathbf{P}\{-2 \le X < 3\}$.

(h) $\mathbf{P}\{-1 \le X \le 3\}$.

6. Suppose that the lifetime X (in years) of a particular television model is exponentially distributed with parameter $\lambda = 1/2$ so that the density function of X is

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2}, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

Suppose that three televisions of this particular model are selected at random, and assume that television lifetimes are independent.

(a) Determine the probability that all three televisions last for at least two years.

(b) Determine the probability that exactly one television lasts for less than one year (so that the other two last for at least one year).

7. Toyota vehicles have been scrutinized lately because of the surge in recalls that Toyota have announced. Suppose that cars account for 65% of Toyota's vehicle production, trucks account for 20% of Toyota's vehicle production, and vans account for the remaining 15% of their vehicle production. Suppose further that 10% of Toyota cars are recalled, 8% of Toyota trucks are recalled, and 12% of Toyota vans are recalled. Assuming that vehicles are recalled independently of other vehicles, what is the probability that a randomly selected recalled vehicle is a truck? *Note: In order to receive full points, you must answer this question by carefully defining events A, B_1, B_2, and B_3 and applying Bayes' Rule symbolically. You many use a tree diagram for motivation and intuition, but your written solution needs to be symbolic.*