Stat 160 Fall 2008
Solutions to Assignment #10

23.16 (a) Let p_j denote the proportion of births on the jth day of the week (with Sunday corresponding to day 1). If all days were equally likely, we would have $p_1 = p_2 = \cdots = p_7 = 1/7$ so that the null hypothesis is

$$H_0 : p_1 = p_2 = \cdots = p_7 = \frac{1}{7}.$$

The alternative is given by

$$H_a : \text{not all } p_j \text{ are equal.}$$

Since there are 700 data points given, assuming H_0 is true, we would expect 100 births on each day.

23.16 (b) The chi-square statistic is

$$\begin{aligned} X^2 &= \frac{(84-100)^2}{100} + \frac{(110-100)^2}{100} + \frac{(124-100)^2}{100} + \frac{(104-100)^2}{100} \\ &\quad + \frac{(94-100)^2}{100} + \frac{(112-100)^2}{100} + \frac{(72-100)^2}{100} \\ &= 19.12. \end{aligned}$$

23.16 (c) There are $df = 7 - 1 = 6$ degrees of freedom, so we see from Table E that $0.0025 < P < 0.005$. Thus, we have strong evidence that births are not spread evenly across the week.

23.18 The question that we wish to answer is the following. Does the GSS data suggest that births are not spread uniformly across the year? Let p_j denote the proportion of individuals having the jth astrological sign (with Aries corresponding to sign $j = 1$). If all astrological signs were equally likely, we would have $p_1 = p_2 = \cdots = p_{12} = 1/12$ so that the null hypothesis is

$$H_0 : p_1 = p_2 = \cdots = p_{12} = \frac{1}{12}.$$

The alternative is given by

$$H_a : \text{not all } p_j \text{ are equal.}$$

Since there are 2779 data points given, assuming H_0 is true, we would expect $231.58\overline{3}$ individuals having each sign. The chi-square statistic is given by

$$\begin{aligned} X^2 &= \frac{(225-231.58\overline{3})^2}{231.58\overline{3}} + \frac{(222-231.58\overline{3})^2}{231.58\overline{3}} + \frac{(241-231.58\overline{3})^2}{231.58\overline{3}} + \cdots + \frac{(244-231.58\overline{3})^2}{231.58\overline{3}} \\ &= 14.39. \end{aligned}$$

With degrees of freedom $df = 12 - 1 = 11$, we see from Table E that $0.20 < P < 0.25$. Thus, we do not reject H_0 which suggests that births are spread uniformly throughout the year. (Equivalently, there is not enough evidence to conclude that births are not uniformly spread through the year.)

23.31 (a) Out of 900 adults, there are 578 that would allow a racist to speak. Therefore, a 99% confidence interval using the plus-four method for the true proportion of all adults that would allow a racist to speak is

$$\tilde{p} \pm z^* \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$$

where $\tilde{p} = \frac{578+2}{900+4} = 0.6416$.

Thus, the required confidence interval is given by

$$0.6416 \pm 2.576\sqrt{\frac{(0.6416)(0.3584)}{904}} = [0.6005, 0.6827].$$

23.31 (b) The column percents are given in the following table.

	Black	White	Other
Allowed	$67/120 = 0.5583$	0.6538	0.6731
Not allowed	$53/120 = 0.4417$	0.3462	0.3269

From the table, we see that the proportion of blacks is noticeably lower than the white and other proportions (which are quite similar). We want to test the hypothesis

$$H_0 : p_b = p_w = p_o$$

against

$$H_a : \text{some proportions are different.}$$

The expected counts are given by the following table.

	Black	White	Other
Allowed	77.07	467.54	33.40
Not allowed	42.93	260.46	18.60

(Recall that each cell's entry is given by the corresponding row total times the column total divided by the grand total.) The chi-square statistic is given by

$$\begin{aligned} X^2 = &\frac{(67 - 7707)^2}{77.07} + \frac{(476 - 467.54)^2}{467.54} + \frac{(35 - 33.40)^2}{33.40} \\ &+ \frac{(53 - 42.93)^2}{42.93} + \frac{(252 - 260.46)^2}{260.46} + \frac{(17 - 18.60)^2}{18.60} \\ = &\ 4.319. \end{aligned}$$

There are $df = 2$ degrees of freedom, and so from Table D we find $0.10 < P < 0.15$. There is not enough evidence to conclude that attitudes differ. (As a comment, note that if we form a 2×2 table by combining the "white" and "other" counts, the chi-square test *is* significant ($X^2 = 4.241$, $df = 1$, $P > 0.025$). In some situations, there may be good justification for this kind of aggregation, but we should be cautious about doing so.)

23.32 The appropriate two-way table is shown below. The two categorical variables are "cocaine use" and "survey method."

	Yes to cocaine	No to cocaine	TOTAL
Phone	168	632	800
One-on-one	200	600	800
Anonymous	224	576	800
TOTAL	592	1808	2400

(Note that in order to fill in this table, we need to work backwards from the proportions given in the problem.) Let p_{ph} be the true proportion of phone respondents who use cocaine, let p_{one} denote the true proportion of personal interview respondents who use cocaine, and let p_{anon} denote the true proportion of anonymous survey respondents who use cocaine. We want to test the hypothesis

$$H_0 : p_{\text{ph}} = p_{\text{one}} = p_{\text{anon}}$$

against

$$H_a : \text{some proportions are different.}$$

The expected counts are given by the following table.

	Yes to cocaine	No to cocaine
Phone	$197.\overline{3}$	$602.\overline{6}$
One-on-one	$197.\overline{3}$	$602.\overline{6}$
Anonymous	$197.\overline{3}$	$602.\overline{6}$

(Recall that each cell's entry is given by the corresponding row total times the column total divided by the grand total.) The chi-square test statistic is given by

$$\begin{aligned} X^2 &= \frac{(168 - 197.\overline{3})^2}{197.\overline{3}} + \frac{(200 - 197.\overline{3})^2}{197.\overline{3}} + \frac{(224 - 197.\overline{3})^2}{197.\overline{3}} \\ &\quad + \frac{(632 - 602.\overline{6})^2}{602.\overline{6}} + \frac{(600 - 602.\overline{6})^2}{602.\overline{6}} + \frac{(576 - 602.\overline{6})^2}{602.\overline{6}} \\ &= 10.619. \end{aligned}$$

There are 3 categories so that the degrees of freedom are $df = 2$. From Table E we see that $0.0025 < P < 0.005$. This is strong evidence that the survey method makes a difference in response.

25.13 (a) The ratio of the largest sample standard deviation to the smallest sample standard deviation is $5.2/4.2 = 1.24$. Since this ratio is less than 2, it is safe to use the ANOVA procedure.

25.13 (b) The overall mean (or grand mean) is

$$\overline{x} = \frac{37 \cdot 10.2 + 36 \cdot 9.3 + 42 \cdot 10.2}{115} = 9.918,$$

and the mean square for groups is

$$MSG = \frac{37(10.2 - 9.918)^2 + 36(9.3 - 9.918)^2 + 42(10.2 - 9.918)^2}{3 - 1} = 10.016.$$

25.13 (c) The mean square for error is

$$MSE = \frac{(37 - 1) \cdot 4.2^2 + (36 - 1) \cdot 4.5^2 + (42 - 1) \cdot 5.2^2}{115 - 3} = 21.897.$$

25.13 (d) The ANOVA F statistic is given by

$$F = \frac{MSG}{MSE} = \frac{10.016}{21.897} = 0.457.$$

There are $3 - 1 = 2$ numerator degrees of freedom and $115 - 3 = 112$ denominator degrees of freedom. Using Table D with $df = (2, 100)$ we see that $P > 0.100$. Thus, we have no reason to doubt the null hypothesis; that is, there is not enough evidence to conclude that mean weight loss differs between these exercise programs.

25.14 The grand mean is given by

$$\overline{x} = \frac{809 \cdot 2.57 + 1860 \cdot 2.32 + 654 \cdot 2.63 + 883 \cdot 2.51 + 207 \cdot 2.51}{4413} = 2.458,$$

the mean square for groups is given by (with $\overline{x} = 2.458$)

$$\begin{aligned} MSG &= \frac{809(2.57-\overline{x})^2 + 1860(2.32-\overline{x})^2 + 654(2.63-\overline{x})^2 + 883(2.51-\overline{x})^2 + 207(2.51-\overline{x})^2}{5-1} \\ &= 16.966, \end{aligned}$$

and the mean square for error is

$$\begin{aligned} MSE &= \frac{(809-1)\cdot 1.40^2 + (1860-1)\cdot 1.36^2 + (654-1)\cdot 1.32^2 + (883-1)\cdot 1.31^2 + (207-1)\cdot 1.28^2}{4413-5} \\ &= 1.817. \end{aligned}$$

In other words, this is the complete ANOVA table.

Source	*df*	*SS*	*MS*
variation among groups	$I - 1 = 5 - 1$	$SS(\text{among}) = 67.864$	$MSG = SS/df = 16.966$
variation within groups	$N - I = 4413 - 5$	$SS(\text{within}) = 8009.336$	$MSE = SS/df = 1.817$

The F statistic is therefore given by

$$F = \frac{MSG}{MSE} = \frac{16.966}{1.817} = 9.337.$$

There are $5-1=4$ numerator degrees of freedom and $4413-5=4408$ denominator degrees of freedom. Using Table D with $df = (4, 1000)$ we see that $P < 0.001$. This is very significant, but this is not surprising because the sample sizes were very large. The differences might not have practical importance. (The largest difference is 0.31, which is relatively small on a 5-point scale.)