Statistics 151 Fall 2006 (Kozdron) Midterm #2 — Solutions

1. We calculate the sample mean as

$$\overline{X} = \frac{55+61+61+64+65+66}{6} = 62$$

and the sample variance as

$$S^2 = \frac{(55-62)^2+(61-62)^2+(61-62)^2+(64-62)^2+(65-62)^2+(66-62)^2}{6-1}$$
$$= \frac{49+1+1+4+9+16}{5} = \frac{80}{5} = 16.$$

Thus, the sample standard deviation is $S = \sqrt{16} = 4$. Therefore, a 95% confidence interval for μ is given by

$$\overline{X} \pm t_{0.025,5}\frac{S}{\sqrt{n}} \quad \text{or} \quad 62 \pm 2.571 \cdot \frac{4}{\sqrt{6}}.$$

(Note that a t-value is required since σ is unknown and n is less than 30.)

2. (a) Since $(0.630, 0.733)$ is a 95% confidence interval for p with $\hat{p} = 0.681$, and since

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

is the general form of a confidence interval for p, we conclude that

$$0.630 = 0.681 - 1.96\sqrt{\frac{0.681 \cdot 0.319}{n}}$$

Solving for n gives $n = 321$.

(b) The proportion of children respondents from Saskatchewan who were overweight is $\hat{p} = 51/85 = 0.60$. Therefore, a 95% confidence interval for p, the true proportion of children Saskatchewan residents who are overweight is

$$\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{or} \quad 0.60 \pm 1.96\sqrt{\frac{0.60 \cdot 0.40}{85}} \quad \text{or} \quad (0.496, 0.704).$$

(c) Since the 95% confidence intervals for adults $(0.630, 0.733)$ and children $(0.496, 0.704)$ residents of Saskatchewan overlap, there is no significant evidence to suggest that they have different overweight rates.

3. If X denotes the BMD value, then X is normally distributed with mean 0.85 and standard deviation 0.11. Therefore,

$$P(X < 0.74) = P\left(\frac{X - 0.85}{0.11} < \frac{0.74 - 0.85}{0.11}\right) = P(Z < -1) = 0.500 - 0.3413 = 0.1587$$

from Table E.

(a) Therefore, the probability that all 4 of them have BMD values less than 0.74 is

$$\binom{4}{4}(0.1587)^4(0.8413)^0 = (0.1587)^4 = 0.000634.$$

(b) The probability that the average of their 4 BMD values is less than 0.74 is

$$P(\overline{X} < 0.74) = P\left(\frac{\overline{X} - 0.85}{0.11/\sqrt{4}} < \frac{0.74 - 0.85}{0.11/\sqrt{4}}\right) = P(Z < -2) = 0.500 - 0.4772 = 0.0228$$

from Table E.

4. Since any z-based confidence interval for a proportion must be centred at the sample proportion, we conclude that Michael's confidence interval of $(0.75, 0.89)$ must be incorrect since it is not centred at 0.80 (but rather at 0.82).

5. Let μ denote the true difference between scented and unscented scores. Then the appropriate hypotheses for the researcher are:
$H_0 : \mu = 0$,
$H_1 : \mu > 0$.

6. The only correct statement is **(ii)**.