Stat 151.003 Fall 2006 (Kozdron)
Solutions to Practice Exam

1. We calculate $\overline{X} = 377.25$ and $S = 172.07$. Since there are only $n = 12$ data points, and the population standard deviation is unknown, a 90% confidence interval for the mean price of a cellular phone in 1993 is

$$\overline{X} \pm t_{0.05,11}\frac{S}{\sqrt{n}} \quad \text{or} \quad 377.25 \pm 1.796 \cdot \frac{172.07}{\sqrt{12}} \quad \text{or} \quad (288.04, 466.46).$$

2. Let μ denote the true mean bursting pressure of the irrigation company's PVC pipes. The appropriate hypotheses are therefore $H_0 : \mu \geq 350$ vs. $H_1 : \mu < 350$. Since $n = 50$, we conduct a z-test. The z-test statistic is given by

$$z = \frac{\overline{X} - \mu}{S/\sqrt{n}} = \frac{343 - 350}{29/\sqrt{50}} = -1.707.$$

Since this is a one-tailed alternative, the critical value corresponding to $\alpha = 0.05$ is -1.645. Since $z = -1.707 < -1.645$, we reject H_0 and conclude that at the $\alpha = 0.05$ significance level there is evidence to suggest that the mean bursting pressure of this company's pipes is strictly less than 350 psi.

3. If p denotes the true proportion of physicians requesting MRI scans that had an ownership interest in the imaging facility, then based on this sample, a 95% confidence interval for p is

$$\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{or} \quad 0.581 \pm 1.96\sqrt{\frac{0.581 \cdot 0.419}{864}} \quad \text{or} \quad (0.548, 0.614).$$

4. (a) Since there are 25 problems, and 4 choices for each, Mickey is expected to get $25/4 = 6.25$ of them correct.

4. (b) Note that $P(\text{Mickey passes}) = 1 - P(\text{Mickey fails})$. Since

$$\begin{aligned} P(\text{Mickey fails}) &= P(\text{Mickey scores 10 or less}) + P(\text{Mickey scores 11}) + P(\text{Mickey scores 12}) \\ &= 0.97 + \binom{25}{11}(0.25)^{11}0.75^{14} + \binom{25}{12}(0.25)^{12}0.75^{13} \\ &= 0.996 \end{aligned}$$

we conclude $P(\text{Mickey passes}) = 0.004$.

5. This problem is not applicable to Section 003 this semester, and so it will NOT be tested on the common final exam.

6. (a) Since it is stated that the proportion is 53.7% to within 2.0 percentage points, 9 times out of 10, we know that $(0.517, 0.557)$ is a 90% confidence interval for p.

6. (b) Since the general form of a 90% confidence interval for p is

$$\hat{p} \pm 1.645\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

we conclude that

$$0.02 = 1.645\sqrt{\frac{0.537 \cdot 0.463}{n}}$$

which upon solving for n yields $n = 1682.004$ so that the most likely sample size was 1682.

7. In order to check whether or not the data are approximately normally distributed, one should begin by plotting a histogram or stem-and-leaf plot of the data. It should then be evident if the data points in the sample are roughly normally distributed. Unfortunately, the histogram may not reveal approximate normality, in which case it might be necessary to collect more data points. It might be the case that collecting more data points is impractical, or too costly. It might then be necessary to examine whether or not other studies have been conducted on similar data. Those other studies might reveal whether or not normality is a reasonable assumption.

8. (a) Let X denote the size of a person's foot so that X is normally distributed with mean 25 and standard deviation 3. Therefore,

$$P(22 < X < 28) = P\left(\frac{22-25}{3} < \frac{X-25}{3} < \frac{28-25}{3}\right) = P(-1 < Z < 1) = 0.6826 \; (= 0.683)$$

using Table E.

8. (b) If $\overline{X}$ denotes the average size of a person's foot, then X is normally distributed with mean 25 and standard deviation $3/\sqrt{100} = 0.3$. Therefore,

$$P(24.7 < \overline{X} < 25.3) = P\left(\frac{24.7-25}{0.3} < \frac{\overline{X}-25}{0.3} < \frac{25.3-25}{0.3}\right) = P(-1 < Z < 1) = 0.6826 \; (= 0.683)$$

using Table E as in **(a)** above.