Stat 151.003 Fall 2006 (Kozdron)
Solutions to Assignment #4

Page 306 #16: If X denotes the life span of a portable radio, then X is normally distributed with mean 3.1 years and standard deviation 0.9 years. That is, X is $N(3.1, 0.9^2)$. If $\overline{X}$ denotes the mean life span of 47 randomly selected radios, then $\overline{X}$ is normal with mean 3.1 and standard deviation $0.9/\sqrt{47}$. That is,

$$\overline{X} = \frac{X_1 + \cdots + X_{47}}{47} \quad \text{is} \quad N\left(3.1, \frac{0.9^2}{47}\right).$$

Therefore,

$$P(\overline{X} < 2.7) = P\left(\frac{\overline{X} - 3.1}{0.9/\sqrt{47}} < \frac{2.7 - 3.1}{0.9/\sqrt{47}}\right) = P(Z < -3.04) = 0.0012$$

where the last equality followed from Table E.

Page 306 #22: If X denotes the mean systolic blood pressure of a normal adult, then X is normally distributed with mean 120 mm Hg and standard deviation 5.6 mm Hg. That is, X is $N(120, 5.6^2)$.

(a) If one individual is selected at random, then the probability that this individual's pressure is between 120 and 121.8 mm Hg is

$$P(120 < X < 121.8) = P\left(\frac{120 - 120}{5.6} < \frac{X - 120}{5.6} < \frac{121.8 - 120}{5.6}\right) = P(0 < Z < 0.32) = 0.1255$$

where the last equality followed from Table E.

(b) If a sample of 30 adults is randomly selected, and $\overline{X}$ denotes their mean systolic blood pressure, then $\overline{X}$ is normal with mean 120 and standard deviation $5.6/\sqrt{30}$. That is,

$$\overline{X} = \frac{X_1 + \cdots + X_{30}}{30} \quad \text{is} \quad N\left(120, \frac{5.6^2}{30}\right).$$

Therefore, the probability that their mean will be between 120 and 121.8 mm Hg is

$$P(120 < \overline{X} < 121.8) = P\left(\frac{120 - 120}{5.6/\sqrt{30}} < \frac{\overline{X} - 120}{5.6/\sqrt{30}} < \frac{121.8 - 120}{5.6/\sqrt{30}}\right) = P(0 < Z < 1.76) = 0.4608$$

where the last equality followed from Table E.

(c) Notice that the standard deviation for one individual is 5.6 while the standard deviation for the average of 30 individuals is $5.6/\sqrt{30} \approx 1.02$. Thus when the probability for one individual is converted to the standard normal z distribution, the probability that X is between 120 and 121.8 corresponds with the probability that z is positive and is within only 0.32 standard deviations of 0. However, when the probability for the average of 30 individuals is converted to the standard normal z distribution, the probability that $\overline{X}$ is between 120 and 121.8 corresponds with the probability that z is positive and within 1.76 standard deviations of 0. This second probability is clearly significantly larger than the first.

Page 315 #8: If X denotes the average weight of an airline passenger's suitcase, then X is normally distributed with mean 45 pounds and standard deviation 2 pounds. That is, X is $N(45, 2^2)$. Therefore, we must find the value w such that $P(X < w) = 0.85$. Normalizing X we find

$$P(X < w) = P\left(\frac{X-45}{2} < \frac{w-45}{2}\right) = P\left(Z < \frac{w-45}{2}\right) = 0.85.$$

From Table E we find $P(Z < 1.04) = 0.8508$. Thus, we equate $\frac{w-45}{2} = 1.04$ and solve for w so that $w = 1.04 \cdot 2 + 45 = 47.08$ pounds.

Page 315 #10: If X denotes the average cost of XYZ brand running shoes, then X is normally distributed with mean \$83 and standard deviation \$8. That is, X is $N(83, 8^2)$. If a sample of nine pairs of running shoes is randomly selected, and $\overline{X}$ denotes their mean cost, then $\overline{X}$ is normal with mean 83 and standard deviation $8/\sqrt{9}$. That is,

$$\overline{X} = \frac{X_1 + \cdots + X_9}{9} \quad \text{is} \quad N\left(83, \frac{8^2}{9}\right).$$

Therefore, the probability that the mean cost of these nine pairs will be less than \$80 is

$$P(\overline{X} < 80) = P\left(\frac{\overline{X}-83}{8/3} < \frac{80-83}{8/3}\right) = P(Z < -1.125) \approx 0.13.$$

Note that Table E does not give an exact value for $P(Z < -1.125)$. However, we do know that $P(Z < -1.2) = 0.1314$ and that $P(Z < -1.3) = 0.1292$. Thus, we can extrapolate to conclude that $P(Z < -1.125) \approx 0.13$.

Page 332 #14:

(a) We find that a 95% confidence interval for μ, the mean score of all bowlers based on this sample of size 40 is

$$\left(\overline{X} - z_{0.025}\frac{\sigma}{\sqrt{n}}\,,\ \overline{X} + z_{0.025}\frac{\sigma}{\sqrt{n}}\right) \quad \text{or} \quad \left(186 - 1.96 \cdot \frac{6}{\sqrt{40}}\,,\ 186 + 1.96 \cdot \frac{6}{\sqrt{40}}\right)$$
$$\text{or} \quad (184.14,\ 187.86).$$

(b) If there are 100 bowlers used instead of 40, and they produce the same mean of 186, then the 95% confidence interval for μ is

$$\left(\overline{X} - z_{0.025}\frac{\sigma}{\sqrt{n}}\,,\ \overline{X} + z_{0.025}\frac{\sigma}{\sqrt{n}}\right) \quad \text{or} \quad \left(186 - 1.96 \cdot \frac{6}{\sqrt{100}}\,,\ 186 + 1.96 \cdot \frac{6}{\sqrt{100}}\right)$$
$$\text{or} \quad (184.82,\ 187.18).$$

(c) It is not surprising that the confidence interval in (b) is smaller. Although they are both centred at 186, the width of the interval when there are 100 bowlers will necessarily be smaller. This is because the "interval factor" of $z_{0.025}\frac{\sigma}{\sqrt{n}}$ is smaller when n is larger. In fact, there are two things worth noticing here. The first is that for any fixed point estimate $\overline{X}$ and any fixed standard deviation σ, the confidence interval will be narrower for n larger. Secondly, as the sample size increases (i.e., as $n \to \infty$), the width of the confidence interval goes to zero and so the confidence interval approaches $(\overline{X} - 0, \overline{X} + 0)$ or simply $(\overline{X})$. That is, if we sample every individual in the population, then there is no need to form an interval estimate of μ based on $\overline{X}$, since we will know $\overline{X} = \mu$ from our census.

Page 332 #20: If X denotes the length of the growing season for a randomly selected U.S. city, and $\overline{X}$ denotes the mean length for these 35 randomly selected cities, then a 95% confidence interval for μ based on this sample is

$$\left(\overline{X} - z_{0.025}\frac{S}{\sqrt{n}}\ ,\ \ \overline{X} + z_{0.025}\frac{S}{\sqrt{n}}\right) \quad \text{or} \quad \left(190.7 - 1.96 \cdot \frac{54.2}{\sqrt{35}}\ ,\ \ 190.7 + 1.96 \cdot \frac{54.2}{\sqrt{35}}\right)$$

$$\text{or} \quad (172.74,\ 208.66).$$

Note that since we don't know σ, the best we can do is approximate it with the sample standard deviation S.