Stat 151.003 Fall 2006 (Kozdron)
Solutions to Assignment #3

1. **Review Exercise page 223 #10.** Let J be the event that John will purchase a new car, and let M be the event that Mary will purchase a new car. Then we know that $P(J) = 0.39$, $P(M) = 0.73$, and $P(M \text{ and } J) = 0.36$. Therefore, $P(M \text{ or } J) = P(M) + P(J) - P(M \text{ and } J) = 0.39 + 0.73 - 0.36 = 0.76$, and so the probability that neither John nor Mary will purchase a new car is $1 - 0.76 = 0.24$.

Review Exercise page 259 #14. If a success is defined as "ride the train to work," then $p = P(S) = 0.30$. Therefore, if there are $n = 10$ workers, we find

$$P(10 \text{ successes}) = \binom{10}{5}(0.30)^5(0.70)^5 = \frac{10!}{5!(10-5)!}(0.30)^5(0.70)^5 = 0.103.$$

Review Exercise page 259 #18. If a success is defined as "absent from the meeting," then $p = P(S) = 0.10$. Therefore, if there are $n = 50$ members, we find the expected number who will be absent from each meeting to be $n \cdot p = 50 \cdot 0.10 = 5$. The corresponding variance is $n \cdot p \cdot (1-p) = 50 \cdot 0.10 \cdot 0.90 = 4.5$ and so the standard deviation is $\sqrt{4.5} = 2.12$.

2. (a) There are four sample points in this experiment. Drawing a tree diagram, we find that the sample space is $S = \{(MU, RS); (MU, RU); (MS, RU); (MS, RS)\}$ where we have used MS for math successful, MU for math unsuccessful, RS for reading successful, and RU for reading unsuccessful.

2. (b) If R is the event that the reading program is successful, then there are two sample points in R, namely $R = \{(MU, RS); (MS, RS)\}$. If M is the event that the math program is successful, then there are two sample points in M, namely $M = \{(MS, RU); (MS, RS)\}$.

2. (c) There are three sample points in the event "R or M," namely R or M

$$= \{(MU, RS); (MS, RU); (MS, RS)\}.$$

2. (d) There is one sample point in the event "R and M," namely R and $M = \{(MS, RS)\}$.

3. (a) This is a binomial probability problem. If we define "success" as "not registered to vote" then the probability of success is $p = 0.45$. If there are $n = 10$ trials, then the probability of observing exactly 5 successes is

$$P(5) = \binom{10}{5}(0.45)^5(0.55)^5 = \frac{10!}{5!(10-5)!}(0.45)^5(0.55)^5 = 0.2340.$$

3. (b) Using the same notation as in **3. (a)**, we have

$$\begin{aligned} P(2 \text{ or fewer successes}) &= P(0) + P(1) + P(2) \\ &= \binom{10}{0}(0.45)^0(0.55)^{10} + \binom{10}{1}(0.45)^1(0.55)^9 + \binom{10}{2}(0.45)^2(0.55)^8 \\ &= 0.0025 + 0.0207 + 0.0763 \\ &= 0.0995. \end{aligned}$$

4. (a) If 68.26% are painted within 2.5 to 3.5 hours, if 95.44% are painted within 2 to 4 hours, and if painting times X are believed to be normally distributed (with unknown mean μ and unknown variance σ^2), then we know

$$P(2 < X < 4) = 0.9544 \quad \text{and} \quad P(2.5 < X < 3.5) = 0.6826.$$

However, since 95.44% of the area under a bell curve lies within two standard deviations of the mean, and since 68.26% of the area under a bell curve lies within one standard deviation of the mean, we can conclude that if Z is a standard normal then

$$P(-2 < Z < 2) = 0.9544 \quad \text{and} \quad P(-1 < Z < 1) = 0.6826. \tag{1}$$

We can then standardize X as follows:

$$P\left(\frac{2-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{4-\mu}{\sigma}\right) = P\left(\frac{2-\mu}{\sigma} \le Z \le \frac{4-\mu}{\sigma}\right) = 0.9544. \tag{2}$$

Similarly,

$$P\left(\frac{2.5-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{3.5-\mu}{\sigma}\right) = P\left(\frac{2.5-\mu}{\sigma} \le Z \le \frac{3.5-\mu}{\sigma}\right) = 0.6826. \tag{3}$$

By comparing (1) with (2) and (3), we see that we must have

$$\frac{2-\mu}{\sigma} = -2 \text{ and } \frac{4-\mu}{\sigma} = 2$$

as well as

$$\frac{2.5-\mu}{\sigma} = -1 \text{ and } \frac{3.5-\mu}{\sigma} = 1.$$

Solving gives $\mu = 3$.

4. (b) If $\mu = 3$, then since $\frac{4-\mu}{\sigma} = 2$ we find $\sigma = \frac{1}{2}$.

4. (c) The required probability is

$$P(X \le 3) = P\left(\frac{X-3}{1/2} \le \frac{3-3}{1/2}\right) = P(Z \le 0) = 0.500$$

where Z is $N(0,1)$ and Table E was used to calculate the last expression.

4. (d) The required probability is

$$P(2 \le X \le 3) = P\left(\frac{2-3}{1/2} \le \frac{X-3}{1/2} \le \frac{3-3}{1/2}\right) = P(-2 \le Z \le 0) = P(0 \le Z \le 2) = 0.4772$$

where Z is $N(0,1)$ and Table E was used to calculate the last expression.

4. (e) The required probability is

$$P(X \le 2) = P\left(\frac{X-3}{1/2} \le \frac{2-3}{1/2}\right) = P(Z \le -2) = P(Z \ge 2) = 0.500 - 0.4772 = 0.0228$$

where Z is $N(0,1)$ and Table E was used to calculate the last expression.

4. (f) The required probability is

$$P(X > 2) = 1 - P(X \le 2) = 1 - 0.0228 = 0.9772$$

using the result of **4. (e)**.