Math 026L.04 Spring 2002
Test #3

Name: ___________________________________

Read all of the following information before starting the test:

- Be sure that this test has **12** numbered pages.
- There are **7** problems on this test, plus a survey problem at the end, worth a total of **102** points, but will be scored out of **100** points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	S	Total
Score									

1. (*24 points*) Consider $\int_0^1 e^{-ax^2}\,dx$ where $a > 0$ is a constant.

(a) Use a left hand Riemann sum with 2 subintervals to approximate the value of this integral. *(Naturally, your solution will involve the constant a.)*

(b) Use a right hand Riemann sum with 2 subintervals to approximate the value of this integral. *(Naturally, your solution will involve the constant a.)*

(Continued)

Suppose that $a = 1/2$.

(c) Use a left hand Riemann sum with 100 subintervals to approximate the value of this integral. *(Be sure to explicitly write the sum you are using to approximate this definite integral as well as what you entered on your calculator.)*

(d) Use a right hand Riemann sum with 100 subintervals to approximate the value of this integral. *(Be sure to explicitly write the sum you are using to approximate this definite integral as well as what you entered on your calculator.)*

(e) What can you say about the true value of $\int_0^1 e^{-\frac{x^2}{2}}\,dx$? Why?

2. (*24 points*) Compute the following integrals.

(a) $\displaystyle\int x^2 + \frac{1}{\sqrt{x}}\, dx$

(b) $\displaystyle\int x(\ln x)^2\, dx$

(Continued)

(c) $\displaystyle\int \frac{x}{\sqrt{1-4x^2}}\,dx$

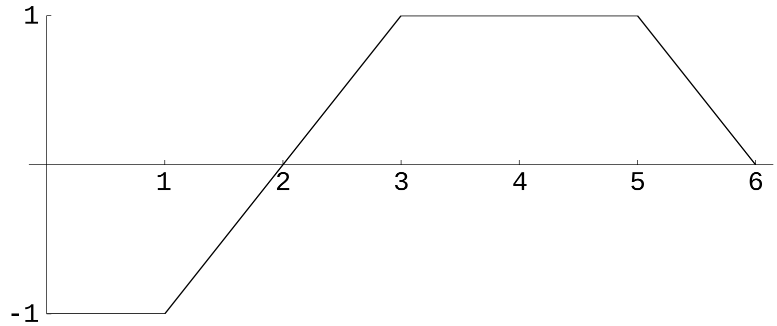
(d) $\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,dx$

3. (*6 points*) Carefully state both parts of the Fundamental Theorem of Calculus.

(a) Part I:

(b) Part II:

4. (*12 points*) Consider the function $f(t)$ whose graph is shown below.



Let the functions $F(x)$ and $g(x)$ be defined on $[0, 6]$ as $F(x) = \int_0^x f(t)\,dt$, and $g(x) = \int_0^x f'(t)\,dt$, respectively.

(a) Compute $F(6)$.

(b) Compute $F'(1)$.

(c) Compute $g(4)$.

(d) Compute $g'(2)$.

5. (*12 points*) Suppose that $g(x)$ is an even function on the interval $[-3, 3]$ and $\int_0^3 g(x)\, dx = 9$. If $\int_{-3}^0 f(x)\, dx = 5$ and $\int_0^3 f(x)\, dx = -3$, answer the following questions.

(a) Evaluate $\int_0^3 2f(x) - 3g(x) + 4\, dx$.

(b) Evaluate $\int_3^{-3} f(x)\, dx$.

(c) Compute the average value of $g(x)$ on the interval $[-3, 3]$.

6. (*12 points*) In the study of probability, the *Cauchy function* is often very useful.

The *Cauchy function*, $P(x)$, is defined for every x by

$$P(x) = \frac{K}{1 + x^2}$$

where K is called the *Cauchy constant.*

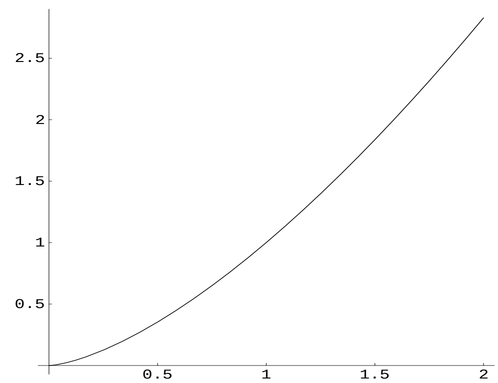
The *Cauchy constant* is chosen so that the value of the definite integral of $P(x)$ from $x = -\infty$ to $x = \infty$ is 1.

Find the *Cauchy constant.* Be sure to justify all of your steps.

(In other words, find the value of K so that $\displaystyle\int_{-\infty}^{\infty} \frac{K}{1 + x^2}\, dx = 1$.*)*

7. (*12 points*) In this problem we will examine the arc length of the function $f(x) = x^{3/2}$ from $x = 0$ to $x = 2$.

(a) Approximate this arc length by hand using two subdivisions. Sketch your approximation on the graph of $f(x)$ provided below.



(b) As you saw in lab, the arc length of a curve $f(x)$ from $x = a$ to $x = b$ is approximated by the sum given below, where Δx_k and Δy_k are the coordinate increments on the k^{th} subinterval. What definite integral does this sum approximate?

$$\sum_{k=1}^{n} \sqrt{1 + \left(\frac{\Delta y_k}{\Delta x_k}\right)^2}\, \Delta x \quad \approx \quad \underline{\hspace{4cm}}$$

(Continued)

(c) Calculate the *exact* arc length of the curve $f(x) = x^{3/2}$ from $x = 0$ to $x = 2$.

Survey Question
(*1 bonus point*)

What did you think of this test? Was it what you were expecting?

Scrap Page

(*You may carefully remove this page from the test booklet.*)