Math 026L.04 Spring 2002
Quiz #6 Solutions

Let $f(t) = \sin(t^2)$.

If the interval is $[2,3]$ and we want 2 subdivisions, then $x_0 = 2$, $x_1 = 2.5$, $x_2 = 3$.

(a) LHS$= \frac{1}{2}f(x_0) + \frac{1}{2}f(x_1) = \frac{1}{2}\left[f(2) + f(2.5)\right] = \frac{1}{2}\left[\sin(2^2) + \sin(2.5^2)\right] \approx -.39499$

(d) RHS$= \frac{1}{2}f(x_1) + \frac{1}{2}f(x_2) = \frac{1}{2}\left[f(2.5) + f(3)\right] = \frac{1}{2}\left[\sin(2.5^2) + \sin(3^2)\right] \approx .18947$

If the interval is $[2,3]$ and we want 10 subdivisions, then

$$\Delta x = \frac{3-2}{10} = \frac{1}{10}$$

and

$$x_0 = 2,\ x_1 = 2 + \tfrac{1}{10},\ x_2 = 2 + \tfrac{2}{10},\ \ldots,\ x_9 = 2 + \tfrac{9}{10},\ x_{10} = 3.$$

(b) LHS$= \frac{1}{10}\left[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9)\right]$

$$= \frac{1}{10}\sum_{k=0}^{9} f\left(2 + \frac{k}{10}\right) = \frac{1}{10}\sum_{k=0}^{9} \sin\left[\left(2 + \frac{k}{10}\right)^2\right] \approx -.09205$$

(e) RHS$= \frac{1}{10}\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10})\right]$

$$= \frac{1}{10}\sum_{k=1}^{10} f\left(2 + \frac{k}{10}\right) = \frac{1}{10}\sum_{k=1}^{10} \sin\left[\left(2 + \frac{k}{10}\right)^2\right] \approx .02484$$

If the interval is $[2,3]$ and we want 100 subdivisions, then

$$\Delta x = \frac{3-2}{100} = \frac{1}{100}$$

and

$$x_0 = 2,\ x_1 = 2 + \tfrac{1}{100},\ x_2 = 2 + \tfrac{2}{100},\ \ldots,\ x_{99} = 2 + \tfrac{9}{100},\ x_{100} = 3.$$

(c) LHS$= \frac{1}{100}\left[f(x_0) + f(x_1) + \cdots + f(x_{98}) + f(x_{99})\right]$

$$= \frac{1}{100}\sum_{k=0}^{99} f\left(2 + \frac{k}{100}\right) = \frac{1}{100}\sum_{k=0}^{99} \sin\left[\left(2 + \frac{k}{100}\right)^2\right] \approx -.03708$$

(f) RHS$= \frac{1}{100}\left[f(x_1) + f(x_2) + \cdots + f(x_{99}) + f(x_{100})\right]$

$$= \frac{1}{100}\sum_{k=1}^{100} f\left(2 + \frac{k}{100}\right) = \frac{1}{100}\sum_{k=1}^{100} \sin\left[\left(2 + \frac{k}{100}\right)^2\right] \approx -.02539$$

(c) As the number of subintervals increases, both the LHS and RHS are decreasing, respectively, and for each n, LHS $<$ RHS. Therefore we suspect that

$$-.03708 < \int_2^3 \sin(t^2)\, dt < -.02539.$$