Math 026L.04 Spring 2002
Assignment #10

This assignment is due Friday, April 19, 2002. You must work through all problems on your own. You may consult any reference materials, and seek help in the Help Room, but do not discuss these problems with anyone else in the class. Answers must be justified whenever possible in order to earn full credit.

1. Do not hand in.

(a) Read Assignment #7 Problem #3 again.

(b) Calculus Page 395 #3

2. Calculus Page 396 #9, #10, #11

3. Consider the function $f(x) = ce^{-3x}$ defined for $0 < x < \infty$.

(a) Find the value of c so that $f(x)$ is a density function.

(b) Sketch the graph of $f(x)$.

(c) Compute the distribution function $F(x)$ for f.

(d) Sketch the graph $F(x)$.

(e) For what value of a does $\displaystyle\int_0^a f(x)\,dx = \frac{1}{2}$. *($a$ is called the median of f.)*

(f) Compute $\displaystyle\int_0^\infty xf(x)\,dx$. *(This value is called the mean of f.)*

4. In the study of probability, the *Beta function* is often very useful.

The *Beta function*, $\beta(x)$, with unknowns r and s is defined on $[0, 1]$ by

$$\beta(x) = Bx^{r-1}(1-x)^{s-1}$$

where B is called the *Beta constant*.

The *Beta constant* is chosen so that the value of the definite integral of $\beta(x)$ from $x = 0$ to $x = 1$ is 1.

Suppose that $r = 3$ and $s = 3$. Find the *Beta constant*.

(That is, find the value of B so that $\displaystyle\int_0^1 Bx^{r-1}(1-x)^{s-1}\,dx = 1$ when $r = 3$ and $s = 3$.)