Math 026L.04 Spring 2002
Assignment #1: Math 25L Review Assignment

This assignment is due at the beginning of class on Friday, January 18, 2002. You may work with others on this assignment, but each student must submit an individual write-up. Since the answers are provided, credit will only be awarded for completely justified solutions. Show all work neatly and in order, and staple your completed assignment before handing it in.

1. For each of the following find $\frac{dy}{dx}$.

(a) $y = x^3 - 6x^2 + 7x - 2$

(b) $y = 3^{2x} - \sqrt{x^2+1}$

(c) $y = \frac{\ln x}{x}$

(d) $y = (x^2 - 9)e^{3x}$

(e) $y^3 - 3xy = -4$

2. Find the equation of the tangent line to the graph of the function $y = (x^2 - 9)e^{3x}$ at $x = 0$.

3. Consider the function $y = \dfrac{\ln x}{x}$.

(a) Find the critical points on the graph of the function and identify them as being a local maximum, a local minimum or neither.

(b) Find the global maximum and global minimum of the function on the interval $[2, 5]$.

(c) Find the inflection point on the graph of the function.

4. Use L'Hôpital's rule to find the following limit.

$$\lim_{x \to 0} \frac{e^{2x} - 2x - 1}{x^2}$$

5. In the table below you will find the record of an animal population at various times (given in years).

time (years)	1	2	3	5	6	8	10	11
population	569	716	901	1428	1797	2846	4508	5674

(a) Use either a log-log graph or a semi-log graph to help you find a function which models this data.

(b) Use the function found in the previous part to find the average rate of change of this population in the first 5 years.

(c) Use the function you found in the first part to find the instantaneous rate of change of this population in year 5. (Include units)

6. In the table below you will find the amount of money a company spent on health insurance for its employees over a 13 year period. Use the information in the table to estimate the rate of increase of this expenditure in 1994. Over this 13 year period is the rate of increase of this expenditure increasing or decreasing? Justify your answer.

Year	1989	1992	1993	1995	1996	1998	1999	2001
Cost (millions of $)	1.40	2.60	2.96	3.6	3.9	4.48	4.76	5.32

7. You have been given the task of designing a number of bus shelters for the city of Durham. Each shelter is in the shape of a large rectangular box with a square top, no floor, and open in the front. The materials for the top cost \$20 per square foot and the materials for the sides cost \$12 per square foot. You are allowed to spend no more than \$5000 per shelter. What dimensions should you make the shelter so that they enclose the largest possible volume? Verify that your answer is the maximum.

8. A woman, 5 feet tall, stands 36 feet in front of a high vertical wall. A car with its headlights on rolls towards her at 3 ft./sec. casting her shadow on the wall behind. How fast is the woman's shadow growing when the car is 12 feet away from her? (Assume that the car's headlights are at ground level.)

9. You are listening to a chemistry lecture and the professor writes the following formula on the board:

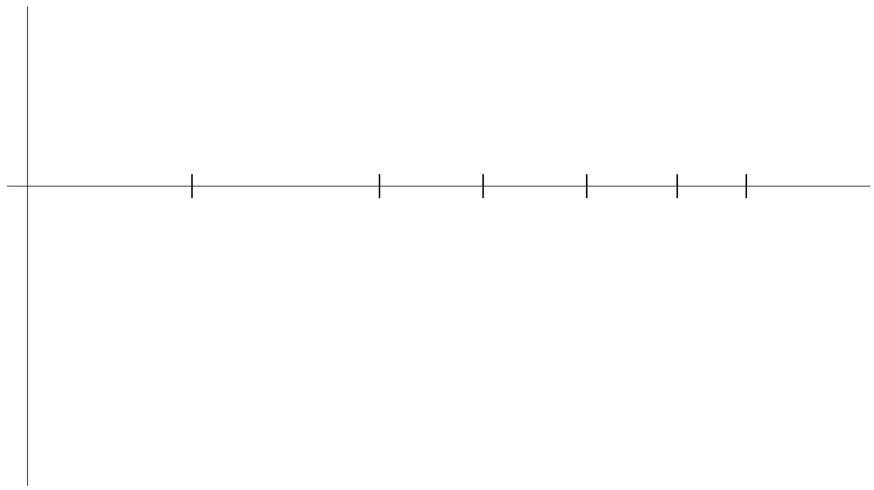
$$S = \frac{e^{-p}}{\sqrt{4-2p}}.$$

She then states that for "small values of p, $S \approx \frac{1}{2} - \frac{3}{8}p$."

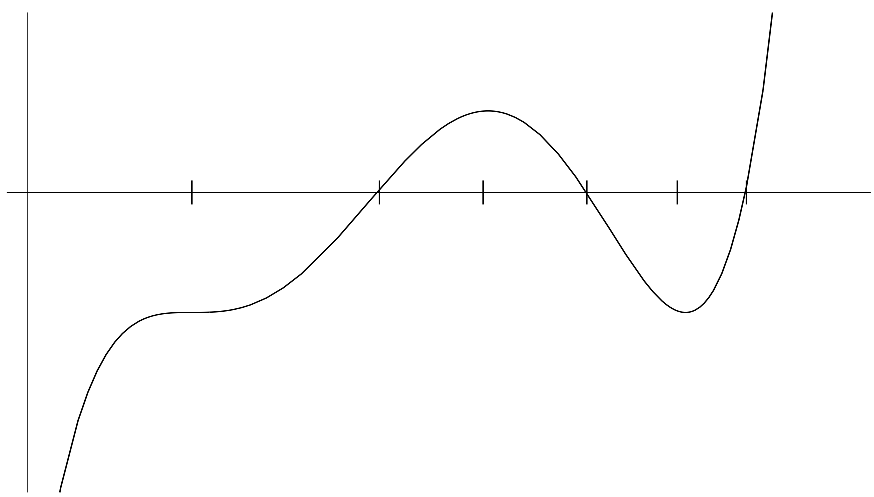
(a) Derive this approximation.

(a) Explain what she meant when she said for "small values of p, $S \approx \frac{1}{2} - \frac{3}{8}p$."

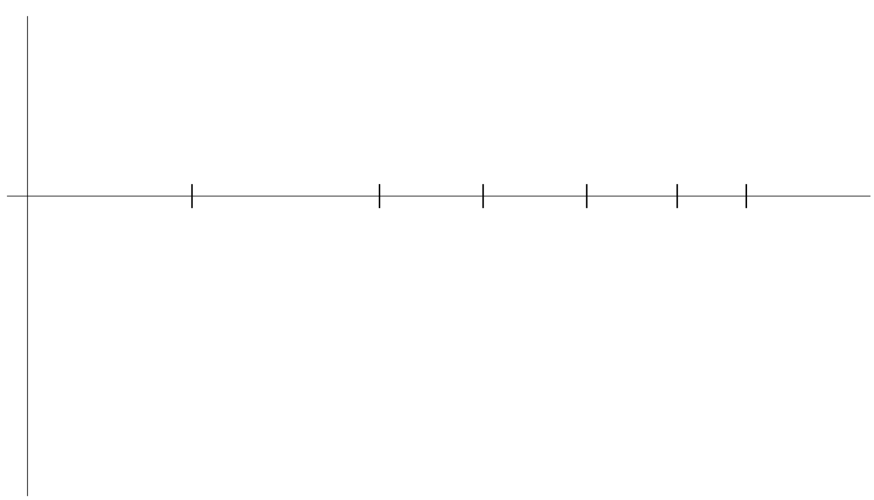
10. On the middle set of axes below is a graph of $f'(x)$, the **derivative** of some function $f(x)$. On the first set of axes sketch $f(x)$ and on the third set of axes sketch $f''(x)$. Be sure that the important features of each graph are clear and that your three graphs are aligned.



$f(x)$



$f'(x)$



$f''(x)$

Answers

(**1a**) $\frac{dy}{dx} = 3x^2 - 12x + 7$

(**1b**) $\frac{dy}{dx} = 3^{2x}(\ln 3)2 - \frac{x}{\sqrt{x^2+1}}$

(**1c**) $\frac{dy}{dx} = \frac{1-\ln x}{x^2}$

(**1d**) $\frac{dy}{dx} = 2xe^{3x} + (x^2 - 9)e^{3x}3$

(**1e**) $\frac{dy}{dx} = \frac{y}{y^2-x}$

(**2**) $y = -27x - 9$

(**3a**) $x = e$ is a local maximum.

(**3b**) $y(e) = \frac{1}{e} \approx 0.3679$ is the global maximum and $y(5) = \frac{\ln 5}{5} \approx 0.3219$ is the global minimum.

(**3c**) $x = e^{\frac{3}{2}}$ is the inflection point.

(**4**) $\lim_{x \to 0} \dfrac{e^{2x} - 2x - 1}{x^2} = 2.$

(**5a**) $P(t) = 452e^{0.23t}$.

(**5b**) 195.1 animals/year.

(**5c**) 328.3 animals/year.

(**6**) $\approx \$333,333$/year. Decreasing.

(**7**) The maximum dimensions are $\frac{5\sqrt{10}}{\sqrt{3}}$ ft. $\times \frac{5\sqrt{10}}{\sqrt{3}}$ ft. $\times \frac{50\sqrt{30}}{27}$ ft. Use the second derivative test to verify this is the maximum.

(**8**) 15/4 ft./sec.

(**9a**) Use a linear approximation for $S(p)$ at 0: $S(p) \approx S(0) + S'(0)p$.

(**9b**) Explain what it means to be a linear approximation for $S(p)$ at 0.