# Math 026L. 01 Spring 2000 <br> Test \#4 

Name: $\qquad$

Read all of the following information before starting the test:

- Be sure that this test has $\mathbf{7}$ pages including this cover.
- There are $\mathbf{4}$ problems on this test worth a total of $\mathbf{5 0}$ points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. No credit will be given for unsupported answers, even if your final answer is correct.
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

| Problem | 1 | 2 | 3 | 4 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Score |  |  |  |  |  |

1. (8 points) A light bulb company is interested in the lifespan of their light bulbs. Suppose that $x$ measures the time (in months) it takes for a light bulb to fail. The company's research indicates that the density function for the lifespan of their light bulbs is well-modelled by

$$
f(x)=\frac{1}{(1+x)^{2}} \text { for } 0 \leq x<\infty
$$

a. (4 pts) What fraction of light bulbs last more than 12 months?
b. (4 pts) What is the median lifespan for these light bulbs?
2. (24 points) Consider the function $f(x)=c e^{-p x}$ defined for $0 \leq x<\infty$ where $p$ is a fixed (but unknown) positive number.
(Note that all of your answers will depend on p.)
a. (6pts) Find the value of $c$ so that $f(x)$ is a density function.
b. ( 6 pts) What is the median of $f(x)$ ?
c. (6 pts) What is the mean of $f(x)$ ?
d. ( 6 pts) Compute the distribution function $F(x)$ for $f$.
3. (8 points) For each of the graphs shown below, decide whether it could represent a distribution function or not. Be sure to justify your answers.
a. (4 pts)

b. (4 pts)

4. (10 points) Suppose that the scores on a certain calculus test were distributed according to the distribution function

$$
F(x)=\frac{2}{\pi} \arcsin (\sqrt{x})
$$

where $0<x<1$ represents the score as a decimal.
a. (4 pts) What is the density function $f(x)$ associated with $F(x)$ ?
b. ( 6 pts) What fraction of students received a score less than 0.60 ?
(Hint: This problem cannot be solved with your answer from a. Think about what information the distribution function gives.)

## Scrap Page

(You may carefully remove this page from the test booklet.)

