Math 026L.01 Spring 2000

Test #3

Name: ________________________________

Read all of the following information before starting the test:

- Be sure that this test has **12** pages including this cover.
- There are **9** problems on this test, plus some bonus problems at the end, worth a total of **100** points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	8	9	B	S	Total
Score												

1. (*8 points*) Suppose that a bacteria culture grows with a constant relative growth rate. Initially the number of bacteria in the culture was 400 and after 6 hours the number was 25 600.

a. (*2 pts*) Write an initial value problem that is satisfied by the number of bacteria in the culture.

b. (*4 pts*) Find an expression for the number of bacteria after t hours.

c. (*2 pts*) When will the number of bacteria reach 100 000?

2. (*8 points*) Solve the initial value problem

$$\frac{x}{\ln x}\frac{dy}{dx} - \frac{1}{y\sin y} = 0, \qquad y(1) = \pi.$$

3. (*16 points*) A tank contains 25 pounds of salt dissolved in 200 gallons of water. There is a pipe at the top of the tank to bring in additional solution and a pipe at the bottom of the tank to drain off solution. The tank itself contains a large stirrer that keeps the solution in the tank thoroughly mixed. Starting at time $t = 0$, water containing water $\frac{1}{2}$ pound of salt per gallon enters the tank at the rate of 4 gallons per minute, and the well-stirred solution leaves the tank at the same rate.

a. (*10 pts*) Find the quantity of salt in the tank as a function of time. *(Be sure to specify a differential equation that describes the rate of change of the amount of salt in the tank and an initial condition for your differential equation.)*

b. (*2 pts*) How much salt is there is the tank after 10 minutes?

c. (*4 pts*) How much salt will be in the tank after a very long time? Does this answer depend on how much salt there was in the tank at the start of the process? Why or why not?

4. (*16 points*) One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor.

a. (*2 pts*) Write a differential equation that is satisfied by y.

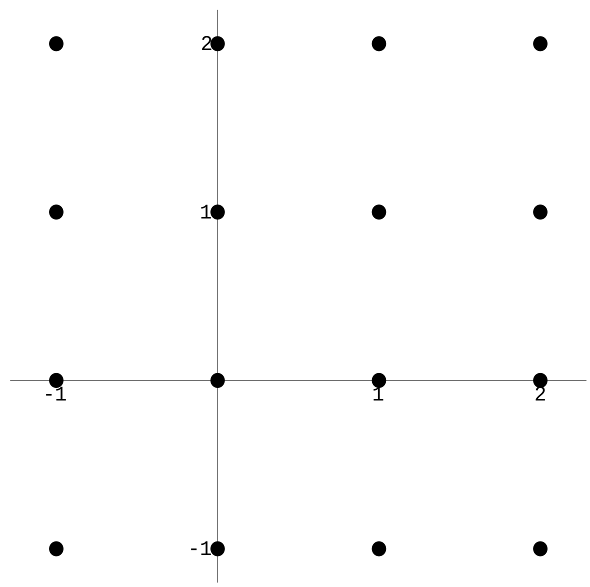
b. (*8 pts*) Solve the differential equation. Assume that initially the fraction y_0 have heard the rumor.

(continued)

c. (*6 pts*) A small town has 1000 inhabitants. At 8 a.m., 80 people have heard a rumor. By noon, half the town has heard it. At what time will 90% of the population have heard the rumor?

5. (*16 points*) Consider the differential equation $\dfrac{dy}{dx} = x - y + 1$.

a. (*8 pts*) Sketch the slope field for this differential equation using the grid below.



b. (*2 pts*) On the slopefield your drew above sketch the solution curve corresponding to the initial value $y(-1) = 1$.

c. (*6 pts*) Suppose that $y(-1) = 1$. Use Euler's method with 3 steps to approximate $y(2)$.

6. (*12 points*) The passenger pigeon was present in the United States in vast numbers until late in the nineteenth century when it was heavily hunted for food and sport and its numbers were drastically reduced. Unfortunately, the passenger pigeon could only breed successfully when present in a large concentration. It was believed that the population of the passenger pigeon could be modelled by the differential equation

$$\frac{dP}{dt} = -P(1000 - P)(2000 - P).$$

a. (*8 pts*) Does this differential equation have any equilibrium solutions? If so, identify each equilibrium solution as either stable or unstable.

b. (*2 pts*) Suppose that the initial population of passenger pigeons was 900. In the long run, does the population become extinct? Why or why not?

c. (*2 pts*) Suppose that the initial population of passenger pigeons was 1100. In the long run, does the population become extinct? Why or why not?

7. (*6 points*) Consider the integral $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$.

a. (*2 pts*) Why is this integral improper?

b. (*4 pts*) What is the value of this integral?

8. (*6 points*) Consider the integral $\int_0^1 \frac{1}{\sqrt{x}}\,dx$.

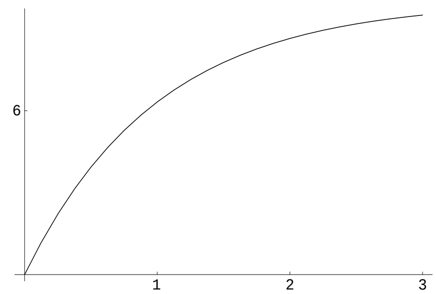
a. (*2 pts*) Why is this integral improper?

b. (*4 pts*) What is the value of this integral?

9. (*12 points*)

a. (*4 pts*) State both parts of the Fundamental Theorem of Calculus.

Consider the function $f(x)$ whose graph is shown below.



Define the functions g and h as follows:

$$g(x) = \int_0^x f(t)\, dt$$
$$h(x) = \int_0^x f'(t)\, dt$$

Compute the following as accurately as possible:

b. (*2 pts*) $g(1)$

c. (*2 pts*) $g'(1)$

d. (*2 pts*) $h(1)$

e. (*2 pts*) $h'(1)$

Bonus Question (*2 bonus points*)

Use the chain rule to compute $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \sin\left(t^2\right)\, dt$.

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?

Scrap Page

(*You may carefully remove this page from the test booklet.*)