Math 026L.01 Spring 2000

Test #2

Name: ___________________________________

Read all of the following information before starting the test:

- Be sure that this test has **11** pages including this cover.
- There are **9** problems on this test, plus some bonus problems at the end, worth a total of **100** points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	8	9	B	S	Total
Score												

1. (*6 points*) On the first test, you proved the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. Using this identity, compute $\int \cos^3\theta \, d\theta$.

2. (*6 points*) Show that $\displaystyle\int \frac{x^2-1}{(x^2+x+1)^2}\,dx = \frac{x^2+1}{x^2+x+1} + C$.

3. (*15 points*) Evaluate the following integrals.

a. (*5 pts*) $\displaystyle\int \tan x \, dx$

b. (*5 pts*) $\displaystyle\int_1^3 x^2 \ln x \, dx$

c. (*5 pts*) $\displaystyle\int \frac{e^x}{1+e^{2x}} \, dx$

4. (*16 points*) Consider $\int_0^4 \frac{1}{1+a^x}\,dx$ where $a \geq 1$ is a constant.

a. (*6 pts*) Use a left hand Riemann sum with 4 subintervals to approximate the value of this integral. *(Naturally, your answer will involve the constant a.)*

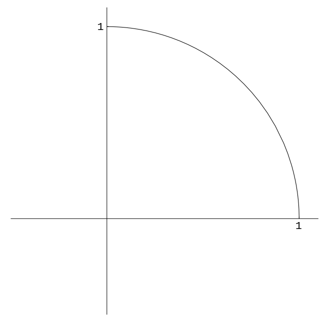
b. (*6 pts*) Use a right hand Riemann sum with 4 subintervals to approximate the value of this integral. *(Naturally, your answer will involve the constant a.)*

c. (*4 pts*) How do the left hand and right hand Riemann sums compare to the true value of $\int_0^4 \frac{1}{1+a^x}\,dx$? Why?

5. (*16 points*)

a. (*3 pts*) State the arc length formula for the length of the arc of the curve $f(x)$ between $x = a$ and $x = b$.

Consider the arc of the unit circle $x^2 + y^2 = 1$ in the first quadrant as shown below.



b. (*3 pts*) Without doing any calculus, write down the length of the arc of the unit circle in the first quadrant.

c. (*10 pts*) Use the arc length formula to determine the length of the arc of the unit circle in the first quadrant, and thus verify your answer from **b.**

6. (*8 points*) In the study of probability, the *Beta function* is often very useful.

The *Beta function*, $\beta(x)$, with unknowns r and s is defined on $[0, 1]$ by

$$\beta(x) = Bx^{r-1}(1-x)^{s-1}$$

where B is called the *Beta constant.*

The *Beta constant* is chosen so that the value of the definite integral of $\beta(x)$ from $x = 0$ to $x = 1$ is 1.

Suppose that $r = 3$ and $s = 3$. Find the *Beta constant.*

(That is, find the value of B so that $\int_0^1 Bx^{r-1}(1-x)^{s-1}\,dx = 1$ when $r = 3$ and $s = 3$.)

7. (*11 points*)

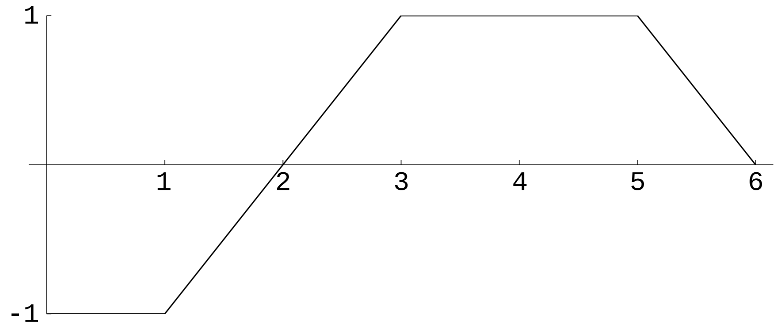
a. (*5 pts*) Write the following series in Σ-notation: $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$

b. (*4 pts*) It is known that if you approximate the value of this series using the first 950 terms, then the approximation is accurate to three decimal places. Use your calculator to find this approximation. *(Be sure to clearly write exactly what you entered on your calculator.)*

c. (*2 pts*) What is $\tan^{-1} 1$ accurate to 3 decimal places? What does this say about the series from **a.**?

(In fact, this series is called the "Leibniz formula for π.")

8. (*10 points*) Consider the function $f(x)$ whose graph is shown below.



a. (*8 pts*) If $F'(x) = f(x)$ and $F(0) = 0$, find $F(b)$ for $b = 2, 4, 6$.

b. (*2 pts*) What is the average value of $f(x)$ on $[0, 6]$?

9. (*12 points*) Suppose that $f(x)$ and $g(x)$ satisfy $f(0) = 5$, $f(1) = 3$, $f(2) = 2$, and $g(0) = 2$, $g(1) = 0$, $g(2) = 3$, respectively.

a. (*6 pts*) Evaluate $\int_0^1 f'(g(x))g'(x)\, dx$.

b. (*6 pts*) Evaluate $\int_0^1 f(x)g'(x)\, dx + \int_0^1 f'(x)g(x)\, dx$.

Bonus Question (*2 bonus points*)

State the Fundamental Theorem of Calculus.

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?

Scrap Page

(*You may carefully remove this page from the test booklet.*)