Math 026L.01 Spring 2000

Test #1

Name: _____

Read all of the following information before starting the test:

- Be sure that this test has **11** pages including this cover.
- There are **9** problems on this test, plus some bonus problems at the end, worth a total of **100** points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	8	9	B	S	Total
Score												

1. (*12 points*) Consider the following trigonometric identities:

- $\cos^2\theta + \sin^2\theta = 1$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Using **only** these identities, prove the following:

a. (*6 pts*) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

b. (*6 pts*) $(\cos\theta + \sin\theta)^2 = 1 + \sin 2\theta$

2. (*10 points*)

a. (*7 pts*) Use the definition of derivative to verify directly that the derivative of

$$f(x) = \cos x \quad \text{is} \quad f'(x) = -\sin x.$$

(No credit will be given for using any "rules of derivatives.")

b. (*3 pts*) Use your result from **a.** to calculate $\dfrac{d}{dx}\sec x$.

(This time you may use any "rules of derivatives.")

3. (*12 points*)

a. (*6 pts*) A bacteria culture starts with 1000 bacteria and after two hours there are 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after six hours.

b. (*6 pts*) Another bacteria culture initially starts with an unknown number of bacteria. The culture is measured after one hour and it is found to contain 2000 bacteria. After two hours, the culture contains 3500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the original number of bacteria in the culture.

4. (*14 points*) Recall that *Newton's Law of Cooling* states that the rate of cooling of an object is proportional to the difference between the temperature of the object and the surrounding temperature.

a. (*7 pts*) A thermometer is taken from a room where the temperature is 20°C to the outdoors, where the temperature is 5°C. After one minute the temperature reads 12°C. What will the temperature be after one more minute?

b. (*7 pts*) On a hot day a thermometer is taken outside from an air-conditioned room where the temperature is 21°C. After one minute it reads 27°C and after two minutes it reads 30°C. What is the outdoor temperature?

5. (*13 points*) Find the **exact** value of the following expressions.

a. (*2 pts*) $\sec \frac{\pi}{4}$

b. (*2 pts*) $\sin\left(-\frac{5\pi}{6}\right)$

c. (*2 pts*) $\tan^{-1}\sqrt{3}$

d. (*2 pts*) $\tan^{-1}(\tan(\sin(\sin^{-1}1)))$

e. (*2 pts*) $\tan(\cos^{-1}0.5)$

f. (*3 pts*) $\cot(\sec^{-1}x)$

6. (*17 points*) If the equation of motion of a particle is given by $s(t) = A\cos(\omega t + k)$, where A, ω, and k are non-zero constants, then the particle is said to undergo *simple harmonic motion.*

a. (*3 pts*) Find $v(t)$, the velocity of the particle at time t.

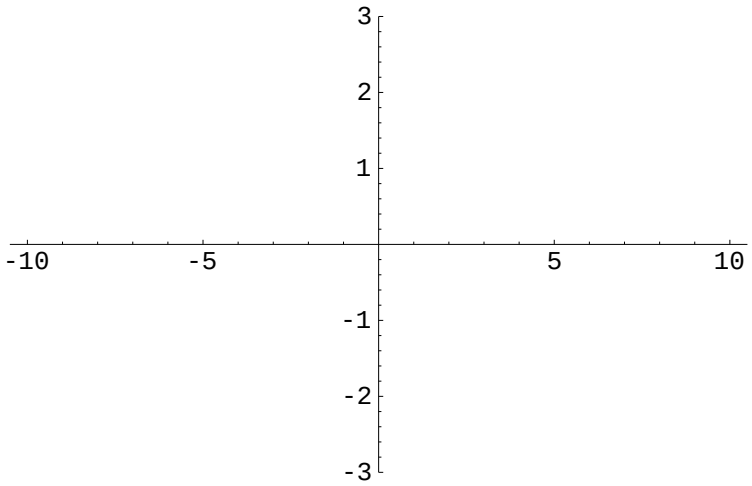
b. (*3 pts*) Find $a(t)$, the acceleration of the particle at time t.

c. (*3 pts*) Verify that $s(t)$ is a solution to the differential equation $\dfrac{d^2s(t)}{dt^2} = -\omega^2 s(t)$.

For **d.** and **e.** suppose that $A = 2$, $\omega = \frac{1}{2}$, and $k = \pi$.

d. (*3 pts*) Find two exact values of t for which the velocity is 0.

e. (*5 pts*) Sketch a graph of $s(t)$ on the axes provided.

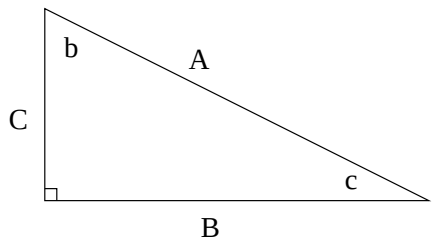


7. (*6 points*) The top of a 200 foot vertical tower is to be anchored by cables that make an angle of 30° with the ground.

a. (*3 pts*) How long must the cables be?

b. (*3 pts*) How far from the base of the tower should anchors be placed?

8. (*6 points*) Consider the right triangle with sides A, B, and C shown below.



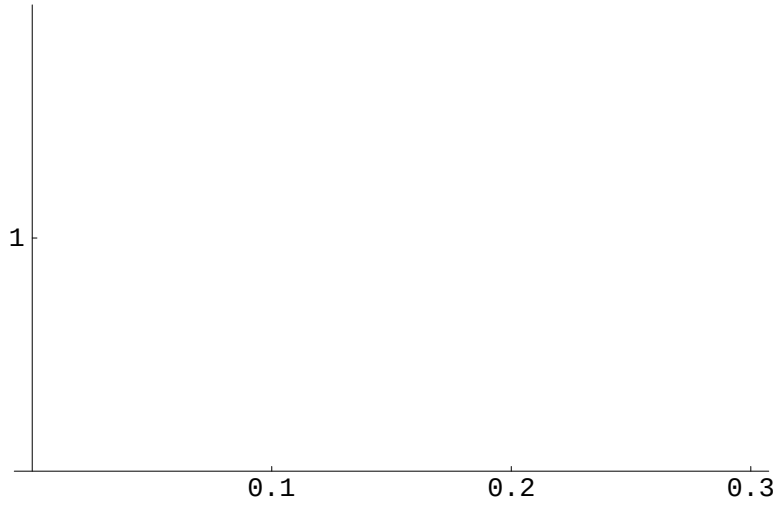
If angle $c = 25°$ and side $B = 3$, solve the triangle by finding angle b and sides A and C.

9. (*10 points*) Suppose $\dfrac{dy}{dx} = -\dfrac{x}{y}$ and $y(0) = 1$.

a. (*5 pts*) Use Euler's method with 3 steps to approximate $y(0.3)$ (to two decimal places).

b. (*3 pts*) Is your approximate value of $y(0.3)$ from **a.** greater than or less than the true value of $y(0.3)$? Explain.

c. (*2 pts*) Sketch an approximation of the graph of $y(x)$ over the interval $[0, 0.3]$.



Bonus Question (*2 bonus points*)

Use the chain rule to show that if θ is measured in degrees, then

$$\frac{d}{d\theta}\sin\theta = \frac{\pi}{180}\cos\theta.$$

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?

Scrap Page

(*You may carefully remove this page from the test booklet.*)