The Definition of the Sine Function

The function $\sin\theta$ is a function assigning to each angle θ some real number $\sin\theta$. In the case when θ is an acute angle, we can define $\sin\theta$ as the ratio of the side lengths of an associated right triangle, as follows. Given an acute angle θ, consider the right triangle that has θ as one of its angles. The value of $\sin\theta$ is then defined to be the ratio of the length of the side opposite to θ to the length of the hypotenuse.

If the angle θ is not an acute angle, the definition above will not work; there will be no right triangle including θ as an angle. We can make a more general definition of $\sin\theta$ as follows. Consider the angle θ as a point on the unit circle, measuring counterclockwise from the positive x-axis. We then define $\sin\theta$ to be y-coordinate of this point.

In the case when θ is an acute angle, these two definitions coincide. Given an acute angle θ, the point P on the unit circle corresponding to θ will be in the first quadrant. Consider the right triangle formed by the segment between the origin and P, the vertical segment between P and the x-axis, and the horizontal segment from there to the origin. This right triangle contains θ as one of its angles, so we can use it to define $\sin\theta$ as in the first definition above. The length of the hypotenuse of this triangle is 1 (since P is on the unit circle), and the length of the side opposite to θ is given by the y-coordinate of P. Thus the ratio of the length of the opposite side to the length of the hypotenuse is equal to the y-coordinate of the point on the unit circle corresponding to θ, so the two definitions coincide.