Math 026L Spring 2000
Paragraph Assignment Key, Week 6
Riemann Sums

The area under the curve $f(x)$, above the x-axis, and between $x = a$ and $x = b$ can be approximated by the areas of rectangles since the area of a rectangle is easy to calculate; it is simply the height times the width of the rectangle. In order to approximate the area under $f(x)$, we can subdivide the interval $[a, b]$ into pieces and on each subinterval calculate the area of an appropriate rectangle. A Riemann sum is a sum of areas of rectangles where the appropriate rectangle has a width which equals the width of the subinterval and height $f(x)$ for some value of x in the subinterval. Some convenient choices for the height are the value of the function at the left endpoint of the subinterval, the value of the function at the right endpoint of the subinterval, or the value of the function at the midpoint of the subinterval.

If $f(x)$ is decreasing, then clearly the value of the function at the left endpoint of any subinterval will be greater than the value of the function anywhere else on that interval so that the LHS will over-approximate the true area. On the other hand, the value of the function at the right endpoint of any subinterval will be less than the value of the function anywhere else on that interval so that the RHS will under-approximate the true area.

As the number of subintervals increases, then both the LHS and the RHS become closer and closer to the true area, although it is necessarily the case that RHS $<$ true area $<$ LHS.