Lab Review Names: ______________________________
"Sample Lab Test" ______________________________
26L Spring 2000 ______________________________

Work on whatever problems you have most difficulty with during the lab period. Whatever you do not finish in lab you should do, *as a group*, on your own time. The entire Sample Lab Test is due at the next lab (hand in one copy for your group). As usual, splitting up group work is considered academic dishonesty and will be treated as such; make sure that *each* member of your group contributes to *each* problem. We reserve the right to have any one of you do any one of these problems on the board if we suspect otherwise.

DISCLAIMER: This Sample Lab Test consists of one problem from each of the labs we have done this semester. It is not, however, a complete and total list of all the things you need to know from the laboratory portion of the course. Future tests and the Final Exam will include problems from the labs that are not represented here.

1. Euler's Method

Suppose that you are given an Initial Value Problem, *i.e.* you are given an expression for the derivative $\frac{dy}{dx}$ of some function $y(x)$ and one value of $y(x)$. Euler's method generates a sequence of points (x_k, y_k) with the property that the slope of the line connecting (x_k, y_k) and (x_{k+1}, y_{k+1}) is given by the value of the derivative at the point (x_k, y_k). Given this information, show that $y_{k+1} = y_k + y'_k\,\Delta x$.

2. Solving Triangles

A jet takes off at a 10° angle and travels at a speed of 285 ft/sec. Approximately how long does it take for the jet to reach an altitude of 15,000 feet?

3. Daylight Hours

Find a sine function $r(t)$ that has the following properties:

- $r(t)$ has a minimum value of 0.02 and a maximum value of 0.06.
- The period of $r(t)$ is 3.
- $r(0) = 0.04$.
- $r(t)$ is increasing when $t = 0$.

It may help to sketch a graph of $r(t)$ first. Show how you arrive at your equation for $r(t)$.

4. Riemann Sums

Suppose you are interested in approximating the area $\int_a^b f(x)\,dx$ under the graph of a function $f(x)$ on the interval $[a, b]$. If you used the Riemman sum

$$\sum_{k=0}^{52} f\left(\frac{2x_{k+1} + x_k}{3}\right) \Delta x,$$

describe the "rule" you would be using on each subinterval $[x_k, x_{k+1}]$ to approximate the area under the curve on that subinterval. (Hint: it is *not* one of the rules we have typically used.) How many boxes would you be using?

5. Arc Length

In the arc length lab you saw that you could approximate the length of a curve $y(x)$ over an interval by subdividing the interval into, say, N equal pieces, approximating the length of $y(x)$ on each subinterval by using the distance formula, and then adding up all these subapproximations. If $x_0, x_1, \ldots, x_N$ are the x-coordinates of the subdivision and $y_0, y_1, \ldots, y_N$ are the corresponding y-coordinates (*i.e.* $y_k = y(x_k)$), this approximation can be written as the sum:

$$\sum_{k=0}^{N-1} \sqrt{(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2}.$$

This sum works fine for approximating the length of the curve, but in order to get an *exact* value for the length we'd like to write this sum so that it looks like a Riemann sum; then we'd be able to use a definite integral to calculate the exact arc length. Use algebra to show that if we define $\Delta x_k = x_{k+1} - x_k$ and $\Delta y_k = y_{k+1} - y_k$, then the sum above can be written in the more desirable form:

$$\sum_{k=0}^{N-1} \sqrt{1 + \left(\frac{\Delta y_k}{\Delta x_k}\right)^2}\, \Delta x.$$

6. Newton's Law of Cooling

Suppose you put a 124° pot of leftover spaghetti into your 44° refrigerator. The data below describes the temperature of the spaghetti at various times over a 6 hour period.

Time (minutes)	0	14	50	120	200	285	360
Temperature (°F)	124	112	89	64	52	47	45

Use log plots to find a function that models this data. (Hint: you may want to graph this data first and see where its asymptote is; remember that exponential functions always have an asymptote at $y = 0$.)

7. Net Worth

Your lab group has decided to start a wombat farm. You manage to find venture capitalists who are willing to give you \$3,000 to use as initial capital. Market insiders tell you that the value of your wombat farm will grow at the same continuous growth rate as the fast-food economy, which varies sinusoidally between 0.02 and 0.06 with a period of 3 years. You plan to start your wombat farm when the continuous growth rate of the fast-food economy is 0.04 and moving up. You have a fixed cost of \$1,500 per year to pay for the land and power to run the farm. In addition, you will incur an operational cost each year that is inversely proportional to the amount of time the wombat farm has been in operation (with proportionality constant 0.12). Write an Initial Value Problem whose solution will be the net worth $W(t)$ of your wombat farm t years after you start it. Don't try to solve or do anything with your IVP, just write it down (and explain how you arrived at your answer).

8. Logistic Growth

Show that the function

$$y(t) = \frac{y_0 M}{(M - y_0)e^{-Mkt} + y_0}$$

is a solution to the logistic growth Initial Value Problem $\dfrac{dy}{dt} = ky(M - y),\ \ y(0) = y_0$ (by "checking").