Most packages of lightbulbs advertise that their lightbulbs have an average life of 800 to 1200 hours (about 33 to 50 days of continuous use). Of course the actual life of a given lightbulb may be more or less than the average life stated on the package. Suppose you buy a lightbulb and install it in the lamp on your desk. What is the likelihood that your lightbulb will burn out after just 100 hours of use? How likely is it that your lightbulb will last for more than 2000 hours? In this lab we will examine such questions, first through discrete data and then through a continuous model.

Part I: Using Lightbulb Data

We begin our investigation of these questions by examining some data describing the length of time that it took each of a sample of 100 lightbulbs to burn out. By figuring out what fraction of the lightbulbs have burned out over a given time interval, we can obtain the probability that a given lightbulb of the same type would burn out in that time interval. We can also use the data to obtain discrete approximations for the mean and median lifetimes of such lightbulbs.

Smith and Moore (of Duke University) did a research study where they bought 100 lightbulbs of the same type and started them burning at the same time. Every day for 200 days they recorded the number of lightbulbs that had burned out already. Here is a subset of the data they obtained:

day	1	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	200
bulbs out	3	20	36	49	60	67	75	79	83	86	89	91	93	95	96	97	100

1. According to the data, what is the probability that a lightbulb of the type tested above will burn out in the first two weeks (of continuous use)? What is the probability that a lightbulb will have burned out in less than 12 weeks? What is the probability that a lightbulb will last at least 8 weeks?

2. Use the data to approximate the average lifetime (in days) of a lightbulb of the type tested (you may have to think for a moment about how to calculate that). What is the approximate median lifetime?

Part II: Modelling the Lightbulb Data

In this Part we will find a function $B(t)$ that models the data from Part I; in other words, we will find a function $B(t)$ that approximates the number of bulbs that have burned out by the end of day t.

1. Make a scatterplot of the data and sketch the graph below. Be sure to use the best window possible and to number and label your axes.

2. By looking at the scatterplot it would be reasonable to guess that the data could be modelled by a shifted exponential function with asymptote at $B = 100$. However, if we shift the data down by 100 units (*i.e.* look at $B - 100$), then the new data will be negative and we won't be able to use log plots (why?). Instead, use log plots to find a function that would model $100 - B$ versus t (do you think that function would be exponential?), and then use this function to obtain a function for $B(t)$.

Part III: Obtaining Distribution and Density Functions

In this part of the lab we will find density and distribution functions for the lightbulb data. Using these functions we can calculate the fraction of lightbulbs that burned out during a given time period, and thus the probability that a given lightbulb will burn out in that time period. We will also find approximations for the mean and median lifetimes of the type of lightbulbs tested.

1. What are the units of $B(t)$? Use your function $B(t)$ to construct a function $F(t)$ that will approximate the *fraction* of lightbulbs that have burned out by the end of day t. (This is easy; think about it.) Note that your function $F(t)$ will only make sense when t is greater than zero; redefine $F(t)$ to be a piecewise function that is equal to what you found above when $t \geq 0$, and zero when $t < 0$.

2. Using your function $F(t)$, estimate the fraction of bulbs that will have burned out in the first ten days. What is the probability that a typical bulb of the same type will burn out in the first ten days? Use $F(t)$ to find the probability that a typical lightbulb will burn out sometime between day 50 and day 100. (Notice that these questions would be difficult to answer using only the data; why?) How can you use $F(t)$ to find the probability that a given lightbulb will burn out between day a and day b?

3. Find $\lim_{t\to\infty} F(t)$ and $\lim_{t\to-\infty} F(t)$. Keep in mind that your function $F(t)$ is only nonzero for positive values of t. Is $F(t)$ a nondecreasing function? What special type of function has these properties (and the relationship described in the question above to the probability that a lighbulb will burn out during a given time period)?

4. What is the relationship between your function $F(t)$ and the density function $f(t)$ that approximates the fraction of bulbs that have burned out per day? Think about the relationship between density functions and cumulative distribution functions, and use the second part of the Fundamental Theorem of Calculus. Do the units of $f(t)$ make sense according to this relationship?

5. Find the density function $f(t)$ that corresponds to your cumulative distribution function $F(t)$. Check that $f(t)$ has the properties required of density functions, *i.e.* check that $f(t)$ is always positive, and that $\int_{-\infty}^{\infty} f(t)\,dt = 1$.

Part IV: Using the Distribution and Density Functions

In this part we will use the cumulative distribution function $F(t)$ and the density function $f(t)$ that you found in Part III to calculate probabilities and approximate the mean and median lifetimes of the type of bulbs tested.

1. Use your density function $f(t)$ to find the probability that a typical lightbulb will burn out sometime between day 50 to day 100. How you would use $f(t)$ to find the probability that a given lightbulb will burn out between day a and day b? Will you get the same answer as when you use the cumulative distribution function $F(t)$ to find this information? Why?

2. Use $f(t)$ to find the mean lifetime of the type of lightbulbs tested. (Recall that if $f(t)$ is a density function for some data, then the mean value of the data is $\int_{-\infty}^{\infty} t\, f(t)\, dt$.) Don't forget that your density function $f(t)$ is zero for negative values of t. Compare your answer to your approximation from Part I.

3. Use $f(t)$ to find the median lifetime of the type of lightbulbs tested. (Recall that if $f(t)$ is a density function for some data, then the median value of the data is the value T for which $\int_{-\infty}^{T} f(t)\, dt = 0.5$.) Again, don't forget that your density function $f(t)$ is zero whenever $t < 0$. Compare your answer to your approximation from Part I.

Assignment: Hand in one *neatly* filled-in copy of this lab for your group. Write your answers on separate paper only for the problems that it is absolutely necessary to do so. If you do not finish by the end of the lab period you may hand the lab in at the beginning of the test on Monday.