Paragraph Assignment, Week 7
Key

Arc Length vs. Area

For a graph of a function f, approximating the area under the graph and approximating the arclength of the graph both involve finding many sub-approximations and adding them up. In other words, to approximate either quantity over an interval from a to b, we chop the interval into smaller subintervals, approximate over each subinterval, and then add these sub-approximations. However, the sub-approximations themselves are computed differently.

When approximating area, we find approximations for the area on each subinterval. This is done by finding the area of a box whose width is the length of each subinterval and whose height is the height of the function at some point in the subinterval. The idea is that this box has an area "close to" the area under the graph on the subinterval, and thus is a good approximation of that area.

On the other hand, when approximating arc length, we must find approximations for the arc length of the graph on each subinterval. We do this by approximating the graph on each subinterval by a line segment which joins the endpoints of the graph on that subinterval. We can then calculate the length of the line segment easily. Given small enough subintervals, the length of this segment will be "close to" the arc length of the graph on the subinterval, and so is a good approximation for that arc length.