Math 026L.01 Spring 2000

Differential Equations
April 7, 2000

1. Consider the differential equation $\frac{dy}{dx} = x - y$.

a. Draw the slope field for this differential equation.

b. On your slope field, sketch the solution curve corresponding to an initial value of $y(0) = 1$. Use your sketch to approximate the value of y when $x = 1$.

c. Use Euler's method with four steps to approximate $y(1)$ if $\frac{dy}{dx} = x - y$, $y(0) = 1$.

d. Which approximation is better, the one from part **b.** or the one from part **c.** ?

2. Solve the following differential equations:

a. $\frac{dy}{dx} = \frac{x^2+1}{y}$

b. $\frac{dy}{dx} = \frac{y}{x^2+1}$

c. $\frac{dy}{dx} = \frac{1}{\sqrt{xy}}$

d. $\frac{dy}{dx} = xe^y \ln x$

e. $\frac{dy}{dx} = \frac{x^2 e^x \sqrt{y^2-1}}{y}$

f. $\frac{dy}{dx} = -3y(2-y)$

3. Calculus Section 10.5 Page 519 #5

4. Calculus Section 10.5 Page 519 #20

5. Calculus Section 10.6 Page 529 #15

6. Calculus Section 10.6 Page 529 #17

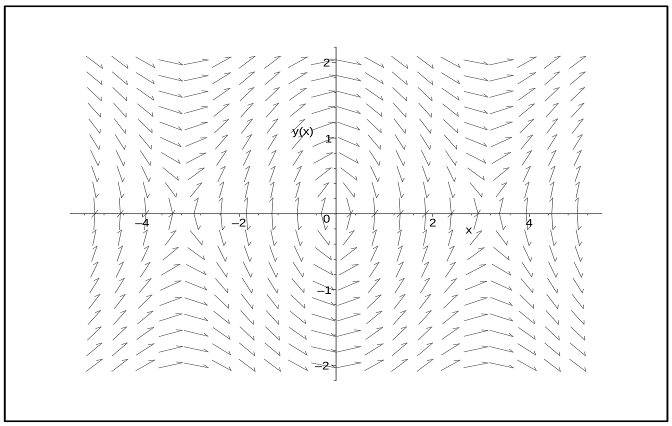
7. Note that the point $(0, 2)$ is on the graph of each of the following three equations:

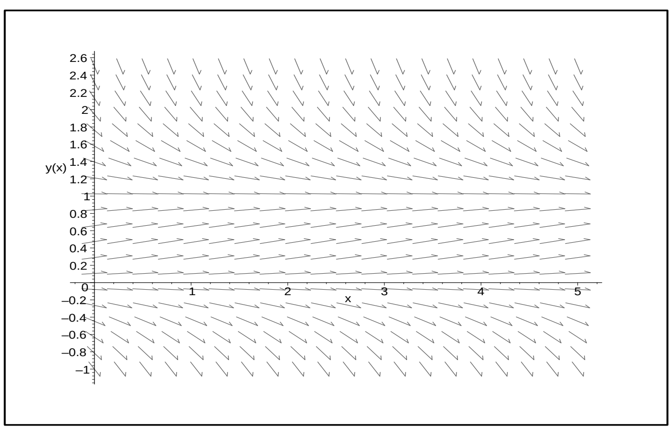
a. $y^2 - 2\cos x = 2$

b. $x \sin y + y = 2$

c. $\ln\left|\frac{y}{1-y}\right| = 0.71x + \ln 2$

Following are slope fields for two of the three equations. Identify which two equations have these slope fields. Label each graph with the letter (**a.**, **b.**, or **c.**) and explain why you made each choice. On each graph, sketch the curve described by the appropriate equation (**a.**, **b.**, or **c.**) that goes through the point $(0, 2)$.





8. Similar to Calculus Section 10.7 Page 539 #5

The growth of a certain animal population is governed by the equation

$$\frac{1000}{P}\frac{dP}{dt} = 100 - P$$

where $P(t)$ is the number of individuals in the colony at time t.

a. What is the carrying capacity of this colony?

b. Sketch a slope field for this differential equation.

c. Suppose that the initial population is known to be 200 individuals. On your slope field sketch the solution curve corresponding to to this inital population.

d. Will there ever be more than 200 individuals in the colony? Will there ever be fewer than 100 individuals? Explain.

e. Solve the differential equation for $P(t)$ assuming $P(0) = 200$.

9. The table gives the number of yeast cells in a new laboratory culture.

Time (hours)	0	2	4	6	8	10	12	14	16	18
Yeast cells	18	39	80	171	336	509	597	640	664	672

a. Plot the data and use the plot to estimate the carrying capacity for the yeast population.

b. Use the data to estimate the initial relative growth rate.

c. Find both an exponential model and a logistic model for these data.

d. Compare the predicted values with the observed values, both in a table and with graphs. Comment on how well your models fit the data.

e. Use your logistic model to estimate the number of yeast cells after 7 hours.