Math 026L.01 Spring 2000
Assignment #5

This assignment is due Wednesday, March 29, 2000. You must work through all problems on your own. You may consult any reference materials, and seek help in the Help Room, but do not discuss these problems with anyone else in the class. Answers must be justified whenever possible in order to earn full credit.

This entire assignment concerns the differential equation

$$\frac{dy}{dx} = xy.$$

You will investigate the solutions to this differential equation three ways: sketching approximate graphs of the solutions using slopefields (#1), approximating values of a solution using (#2) Euler's method, and finding an exact solution using (#3) separation of variables. Finally, you will check and compare your answers (#4).

1. Slopefields

   a. Sketch the slopefield for the differential equation using the grid below. Be sure to make your slopefield neat and accurate, and show your calculuations.
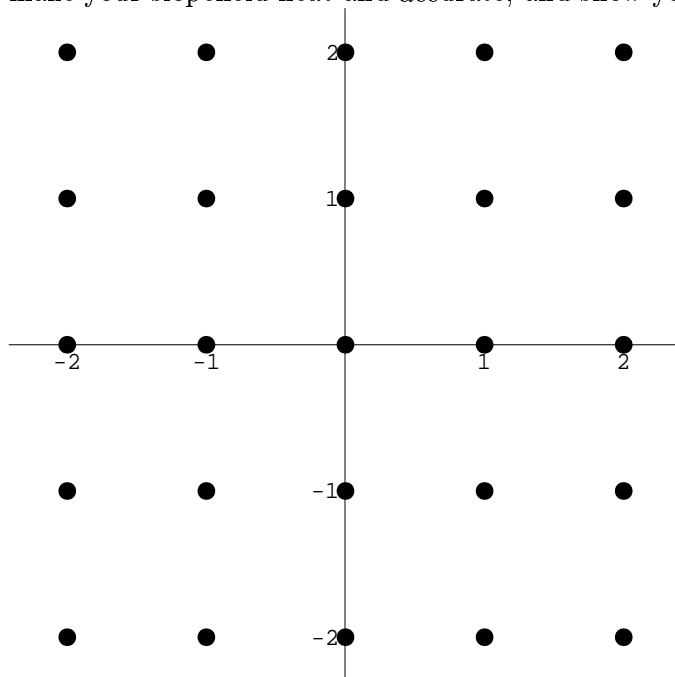
   b. On the slopefield your drew above, use a different color pen to sketch and label solution curves for the following initial values: $y(0) = -1$ and $y(-1) = 1$.

   c. Consider the solution curve that you drew for the initial value $y(-1) = 1$. Try to approximate the value of $y(0)$ for this curve using the graph that you drew (your approximation will not be very accurate).

2.   Euler's Method

For this problem we will only consider the solution that has initial value $y(-1) = 1$. In other words we will consider the Initial Value Problem:

$$\frac{dy}{dx} = xy, \quad y(-1) = 1.$$

a.   Use Euler's Method with $\Delta x = .5$ to approximate the value of $y(0)$. Be sure to show your work and all your calculations clearly.

b.   Use Euler's Method with $\Delta x = .25$ to approximate the value of $y(0)$. Be sure to show your work and all your calculations clearly.

c.   In each of your approximations above, you actually produced a list of points that would be on an approximate sketch of the solution curve $y(x)$. Make an approximate sketch of the solution $y(x)$ on the interval $[-1, 0]$ using the points you obtained in part (a). Then (in a different color pen) sketch the corresponding approximation from part (b). Be sure to label which is which.
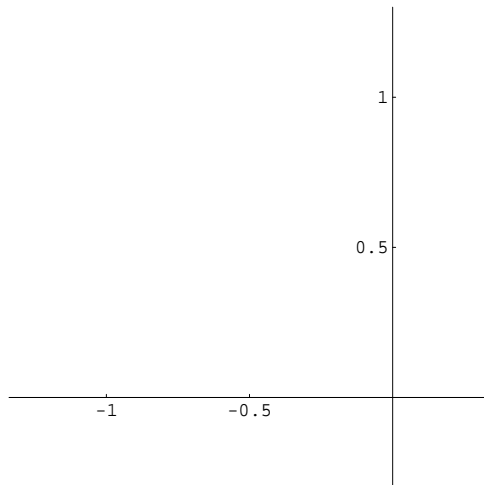
3. Separation of Variables

a. Use separation of variables to find the family of solutions for the differential equation

$$\frac{dy}{dx} = xy.$$

Be sure to show all your work clearly and in order.

b. Find the solutions $y(x)$ that correspond to the initial values $y(0) = -1$ and $y(-1) = 1$.

c. Use your calculator to graph the two solutions you found above. Make a careful sketch of your solutions on the grid below.
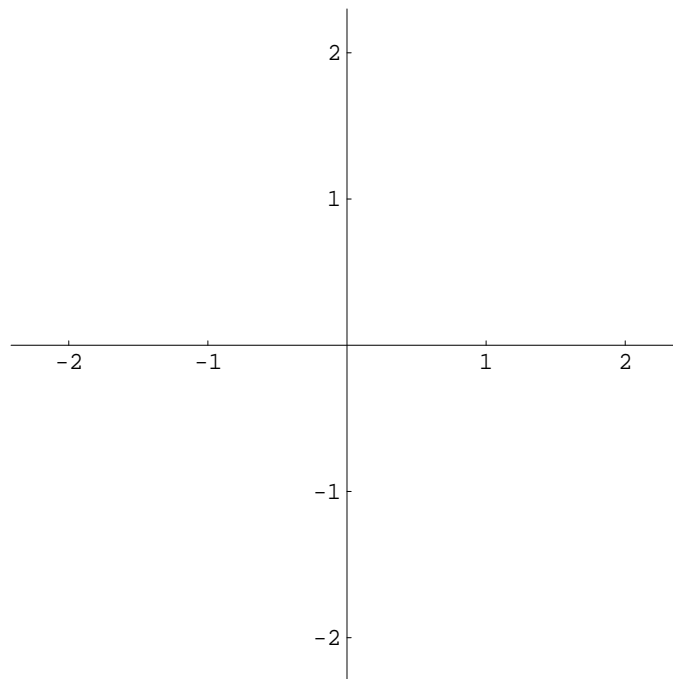
4. Checking

In this problem you will do various checks and comparisons of your answers to problems #1, #2, and #3 above. First let's consider the problems that concerned the IVP

$$\frac{dy}{dx} = xy, \quad y(-1) = 1.$$

a. Which approximation for $y(0)$ should be better, the one found #2a or the one found in #2b? Why?

b. Compare your approximation for $y(0)$ from the slopefield in #1c with your approximation from Euler's Method in #2b. Are they similar? Why/why not? Which do you think is more accurate?

c. Compare the sketches of $y(x)$ that you obtained in #2c from Euler's method with the sketch of $y(x)$ that you drew using the slopefield in #1b. Do they match?

d. Find the actual value of $y(0)$ using the formula for the solution that you found in #3b. Is it close to the approximations of $y(0)$ that you made in #1c, #2a, and #2b?

Now we'll consider the family of solutions to the differential equation

$$\frac{dy}{dx} = xy.$$

e. Compare the graphs of the two exact solution curves from #3c to the curves you drew using the slopefield in #1b. Do they correspond?

f. Check that the family of solutions that you found in #3a is correct. In other words, check that any function in that family of solutions will satisfy the differential equation. (Think back to "Checking Solutions to Differential Equations.")