Math 026L.01 Spring 2000
Assignment #3

This assignment is due at the beginning of class on Monday, February 21, 2000. You must work through all problems on your own. You may consult any reference materials, and seek help in the Help Room, but do not discuss these problems with anyone else in the class. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.

1. (*5 points*) Consider the following trigonometric identities:

- $\cos^2\theta + \sin^2\theta = 1$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

a. (*3 pts*) Using **only** these identities, prove $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1+\tan\alpha\tan\beta}$.

b. (*2 pts*) Use your result from **a.** to show $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

2. (*4 points*)

a. (*2 pts*) Use the chain rule to show that if θ is measured in degrees, then $\frac{d}{d\theta}\sin\theta = \frac{\pi}{180}\cos\theta$.

b. (*2 pts*) Compute $\frac{d}{dx}\csc^{-1}x$.

3. (*6 points*)

a. (*3 pts*) Write the following series in Σ-notation: $\frac{1}{2}+\frac{1}{8}+\frac{1}{24}+\frac{1}{64}+\frac{1}{160}+\cdots$

(Hint: For each term, look for something multiplied by a power of 2.)

b. (*2 pts*) Approximate the value of this series accurate to 3 decimal places. How do you know your approximation is accurate to 3 decimal places? Explain.

c. (*1 pt*) What is $\ln 2$ accurate to 3 decimal places? What does this say about the series from **a.**?

4. (*5 points*) Let $f(x) = 16 - x^2$.

a. (*2 pts*) Use a left hand Riemann sum with 4 subintervals to approximate the area below $f(x)$, above the x-axis, and between $x = 0$ and $x = 1$.

b. (*2 pts*) Use a right hand Riemann sum with 4 subintervals to approximate the area below $f(x)$, above the x-axis, and between $x = 0$ and $x = 1$.

c. (*1 pt*) What can you say about the true area below $f(x)$, above the x-axis, and between $x = 0$ and $x = 1$?