This assignment is due at the beginning of class on Monday, January 31, 2000. You must work through all problems on your own. You may consult any reference materials, and seek help in the Help Room, but do not discuss these problems with anyone else in the class. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.

1. Experiments show that if the chemical reaction

$$N_2O_5 \to 2NO_2 + \frac{1}{2}O_2$$
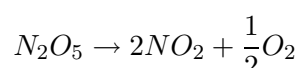
takes place at $45°C$, then the rate of reaction of dinitrogen pentoxide $[N_2O_5]$ is proportional to its concentration, with constant of proportionality -0.0005.

  a. Write a differential equation which models this situation.

  b. Find an expression for the concentration $[N_2O_5]$ after t seconds if the initial concentration is C.

  c. How long will the reaction take to reduce the concentration of $[N_2O_5]$ to 90% of its original value?

2. Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that the difference is not too large. Suppose that the object takes 40 minutes to cool from $30°C$ to $24°C$ in a room that is kept at $20°C$.

  a. What is the temperature of the object 15 minutes after it was $30°C$?

  b. How long would it take the object to cool down to $21°C$?

  c. Suppose that the object is left in the room for a very long time. Eventually, what temperature does the object approach?

  d. Draw a graph of the temperature function. Be sure to illustrate the limiting behaviour from **c.**

3. The population of the island of Little South Dubuque in the West China Sea in 1990 was 1250. Census data taken there indicate that if there is no immigration or emigration, the population grows at a rate proportional to itself, with constant of proportionality .03.

  a. Given these conditions, write and solve an initial value problem for the population function $P(t)$. What does your function predict the popuation is now?

  b. Now suppose that in addition 25 people immigrate to the island every year. Find and solve a new initial value problem using this new assumption. What does this function predict the population is now?

4.   Consider the initial value problem $\dfrac{dy}{dx} = x + y$, $y(0) = 1$.

    a.   Verify that $y = -x - 1$ is NOT a solution to this IVP.

    b.   Use Euler's method with 4 steps to approximate $y(1)$.

    c.   Is your approximation in **a.** an overestimate or an underestimate of the true value of $y(1)$? Explain.