Math 025L.01 Fall 1999

Test #2

1. (*20 points*) For each of the following, decide whether the statement is true or false. If it is true, explain why. If it is false, provide a simple counterexample.

a. (*5 pts*) If $\lim_{x \to 1} f(x)$ exists, then the function f must be defined at 1.

b. (*5 pts*) If $f'' > 0$ and $f > 0$, then f is increasing.

c. (*5 pts*) If $f(x)$ is differentiable at $x = a$, then it is continuous at $x = a$.

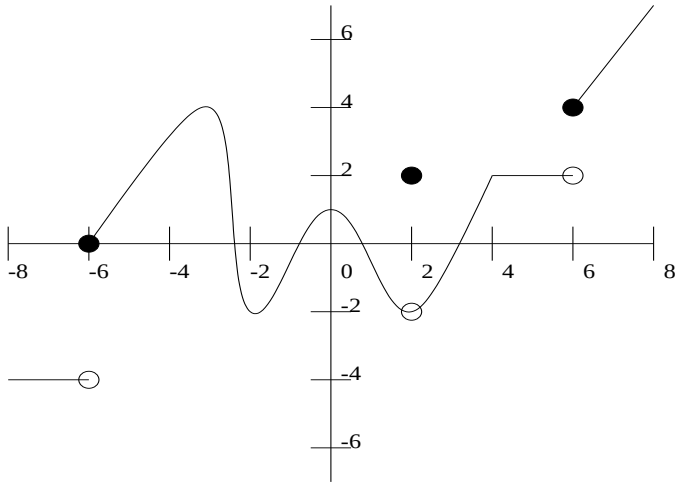
d. (*5 pts*) If $f' < 0$, then $f'' < 0$.

2. (*10 points*) Use the definition of derivative to verify directly that the derivative of

$$f(x) = 2\sqrt{x+1} \quad \text{is} \quad f'(x) = \frac{1}{\sqrt{x+1}}.$$

(No credit will be given for using the "rules of derivatives.")

3. (*10 points*) Use the graph of $g(t)$ pictured here to answer the questions below. *(You do not need to show your work or justify your answers, just fill in the blanks provided.)*



(continued)

a. (*1 pt*) $\lim_{x \to 2} g(t) =$ _____

b. (*1 pt*) $g(2) =$ _____

c. (*1 pt*) $\lim_{x \to 6-} g(t) =$ _____

d. (*1 pt*) $\lim_{x \to 6+} g(t) =$ _____

e. (*1 pt*) $\lim_{x \to -6-} g(t) =$ _____

f. (*1 pt*) $\lim_{x \to -6+} g(t) =$ _____

g. (*2 pts*) $g(t)$ is not continuous at $t =$ _____

h. (*2 pts*) $g(t)$ is not differentiable at $t =$ _____

4. (*10 points*) Let $p(x)$ be a rational function which has

- a double root at $x = b$,
- a horizontal asymptote of 2,
- a vertical asymptote of $x = a$,
- a vertical asymptote of $x = c$.

a. (*4 pts*) Assuming that $0 < a < b < c$, find a formula for $p(x)$.

b. (*6 pts*) Assuming that $0 < a < b < c$, sketch $p(x)$. Be sure to label *all* intercepts.

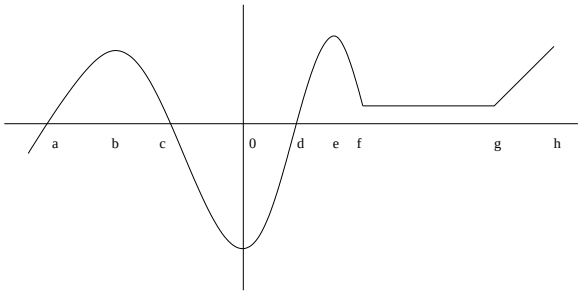
5. (*8 points*)

a. (*2 pts*) Define what it means for a function to be continuous at the point $x = a$.

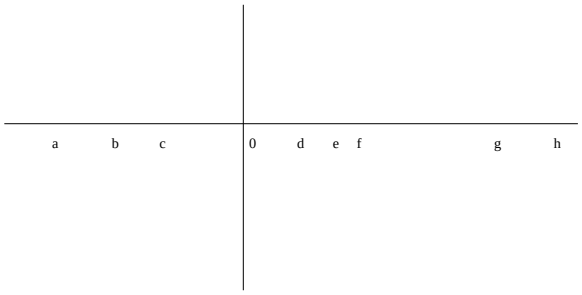
b. (*6 pts*) Let $f(x) = \begin{cases} \dfrac{x^2 - 6x - 7}{x^2 - 8x + 7}, & \text{if } x \neq 7, \\ \dfrac{3}{4}, & \text{if } x = 7. \end{cases}$

Is $f(x)$ continuous at $x = 7$?

6. (*15 points*) Below is a graph of $f'(x)$, the *derivative* of $f(x)$.



a. (*7 pts*) On the set of axes below sketch $f''(x)$.



b. (*4 pts*) Clearly describe where $f(x)$ is concave down.

c. (*4 pts*) Is $f(x)$ continuous for $a < x < h$? Why or why not?

7. (*19 points*) The following table shows the total numbers of AIDS cases in the U.S. reported to the *Centers for Disease Control and Prevention* as of the indicated dates.

Month	Months from Jan 1, 1980	Number of Cases
Sep 1981	20	110
Oct 1981	21	129
Jan 1982	24	220
Mar 1982	26	257
May 1982	28	439
Aug 1982	31	514
Dec 1982	35	878
Mar 1983	38	1,129
Jun 1983	41	1,756
Sep 1983	44	2,057
Dec 1983	47	3,212

Let m be the number of months since January 1, 1980, and let $n = C(m)$ be the number of cases.

a. (*1 pt*) What are the units of $C'(m)$, the derivative of $C(m)$?

b. (*2 pts*) Without performing any calculations, why must $C'(m)$ be positive?

c. (*4 pts*) In terms of month and number of cases, interpret the following:

i. $C(24) = 220$ **ii.** $C^{-1}(3, 212) = 47$

d. (*6 pts*) Using the table of data, estimate $C'(41)$. In practical terms, what does this represent?

e. (*6 pts*) Using the table of data, estimate the derivative of $C^{-1}(n)$ when $n = 257$. In practical terms, what does this represent?

8. (*8 points*) Evaluate the following limits by any appropriate means. *(Be sure to show all your work.)*

a. (*4 pts*) $\lim_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

b. (*4 pts*) $\lim_{x \to 0} x^2 \ln\left(\dfrac{1}{x}\right)$

Bonus Question (*2 bonus points*)

Give an informal argument to explain why $\dfrac{d}{dy} f^{-1}(y) = \dfrac{1}{f'(x)}$ if $y = f(x)$ is invertible and differentiable.

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?