Math 025L.01 Fall 1999

Test #1

1. (*15 points*) For each of the following, decide whether the statement is true or false. If it is true, explain why. If it is false, provide a simple counterexample.

a. (*3 pts*) The inverse of $y = \frac{x+1}{x-2}$ is $y = \frac{x-2}{x+1}$.
b. (*3 pts*) A function is concave up if its average rate of change is increasing.
c. (*3 pts*) The vertical line test is used for checking whether a function is one-to-one.
d. (*3 pts*) Every odd function is invertible.
e. (*3 pts*) Suppose the function $f(x)$ is invertible and that it has a horizontal asymptote. Then its inverse $f^{-1}(x)$ must have a vertical asymptote.

2. (*13 points*) Let A be the set of all people in this room.

a. (*4 pts*) Construct a function from the set A to the set A. (Remember that this means that your domain must be all of A, but that your range does not need to be all of A. It could be a subset of A.)
b. (*4 pts*) Construct a function from the set A to the set A that is one-to-one.
c. (*5 pts*) Is your function in part (a) invertible? Why or why not? Is your function in part (b) invertible? Why or why not?

3. (*15 points*) Consider the function $F(t) = \begin{cases} t^2+1, & \text{if } t \geq 0, \\ t, & \text{if } t < 0. \end{cases}$

a. (*3 pts*) Sketch the graph of $F(t)$.
b. (*1 pt*) Is $F(t)$ an odd function?
c. (*1 pt*) Is $F(t)$ an even function?
d. (*2 pts*) Is $F(t)$ an invertible function? Why or why not?
e. (*2 pts*) What are the domain and range of $F(t)$?

Use the graph of $F(t)$ to sketch the graphs of the following transformations:

f. (*3 pts*) $F\left(-\frac{t}{2}\right)$
g. (*3 pts*) $-2F\left(\frac{t}{2}-1\right)+1$

4. (*10 points*)
a. (*6 pts*) Use your calculator to find all solutions to the equation $2^x = x^2$. Give your answers to one decimal place. Describe what you did to find the solutions. Be sure to sketch the graphs drawn by your calculator as part of the explanation for your answer.
b. (*4 pts*) For what values of x is $2^x > x^2$?

5. (*10 points*) Consider the three functions defined by the tables of data below. One of these tables defines a linear function, one defines an exponential function, and one defines neither a linear nor an exponential function. Say which table corresponds to the linear function and which table corresponds to the exponential function. Then give an equation for the linear function and an equation for the exponential function.

(**a**)

x	0.00	0.50	1.00	1.50	2.00
y	2.25	2.41	2.58	2.76	2.96

(**b**)

x	0.00	0.50	1.00	1.50	2.00
y	3.12	2.62	2.20	1.85	1.55

(**c**)

x	0.00	0.50	1.00	1.50	2.00
y	2.71	3.94	5.17	6.40	7.63

6. (*8 points*) Under ideal conditions, a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria.

a. (*2 pts*) What is the size of the population after 15 hours?

b. (*2 pts*) Find a formula for the population function, $P(t)$, which gives the size of the population after t hours.

c. (*4 pts*) Find $P^{-1}(50,000)$ and explain its meaning in terms of time and population.

7. (*10 points*) The magnitude of an earthquake is measured on the Richter scale. The relationship between the strength, s, of an earthquake and its magnitude, $M(s)$, on the Richter scale is given by the formula $M(s) = \log_{10}\left(\frac{s}{0.001}\right)$.

Find magnitudes on the Richter scale of earthquakes having the following strengths:

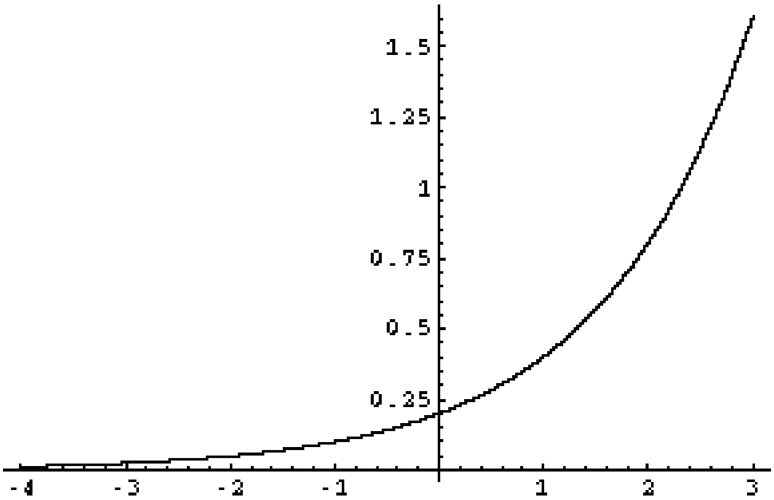
a. (*1 pt*) 1000

b. (*1 pt*) 1,000,000

c. (*4 pts*) The San Francisco earthquake of 1906 measured 8.6 on the Richter scale. Compute the strength of that earthquake.

d. (*4 pts*) The 1994 San Francisco earthquake measured 6.6 on the Richter scale. How many times stronger was the 1906 earthquake than the 1994 earthquake?

8. (*6 points*) Find a possible formula for the exponential function graphed below.



9. (*12 points*) Recall that a power function is a function of the form $f(x) = kx^p$ where k and p are any constants.

a. (*6 pts*) Suppose that $f(x)$ is the power function $f(x) = -3x^2$. What is the average rate of change of $f(x)$ on the interval $a \leq x \leq b$? Simplify your answer as much as possible.

b. (*6 pts*) Let $y = kx^p$ where both k and p are *positive* constants. Show that $\ln(y)$ is a linear function of $\ln(x)$.

Bonus Question (*1 bonus point*)

Two strangers are each asked separately to choose a positive whole numbers, being advised that if they both choose the same number, they both get a prize. If you were one of these people, which number would you choose?

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?