Math 025L.01 Fall 1999
Assignment #9

This assignment is due at the beginning of class on Monday, November 29, 1999.

This is a review assignment and does not test any material that has not yet been covered in class. There are 15 questions that have been given to Math 25L students this semester. All of the questions are appropriate and their solutions require the techniques you will be expected to demonstrate on the actual final.

I will not grade this assignment for correctness, but rather completeness. This is an opportunity for you to practice the skills required of you without the pressure of the final exam. There is no sense in copying someone else's work simply for the sake of completion. That is not in your best interest.

Good luck!

1. (*9 points*) Describe a function from the set of odd integers to the set of even integers, and clearly explain why it is a function.

2. (*12 points*) Recall that a power function is a function of the form $f(x) = kx^p$ where k and p are any constants.

a. (*6 pts*) Suppose that $f(x)$ is the power function $f(x) = -3x^2$. What is the average rate of change of $f(x)$ on the interval $a \leq x \leq b$? Simplify your answer as much as possible.

b. (*6 pts*) Let $y = kx^p$ where both k and p are *positive* constants. Show that $\ln(y)$ is a linear function of $\ln(x)$.

3. (*8 points*)

a. (*2 pts*) Define what it means for a function to be continuous at the point $x = a$.

b. (*6 pts*) Let $f(x) = \begin{cases} \dfrac{x^2 - 6x - 7}{x^2 - 8x + 7}, & \text{if } x \neq 7, \\ \dfrac{3}{4}, & \text{if } x = 7. \end{cases}$

Is $f(x)$ continuous at $x = 7$?

4. (*16 points*) Circle **T** (True) or **F** (False) or give short answers to each of the questions below.

(**a**) **T** **F** $f(x)$ has an inverse if and only if it passes the vertical line test.

(**b**) **T** **F** $\log_a(a) = 1$.

(continued)

(**c**) **T** **F** If $f(x)$ is decreasing and invertible, then $f^{-1}(x)$ is decreasing.

(**d**) **T** **F** If $f(x)$ is concave up and invertible, then $f^{-1}(x)$ is concave down.

(**e**) **T** **F** $e^{2\ln x} = 2$.

(**f**) **T** **F** As $x \to \infty$, $f(x) = 72,000x^{12}$ dominates $g(x) = 10(2^x)$.

(**g**) **T** **F** The function $f(x) = x^x$ is an exponential function.

h. (*3 pts*) If a function $f(x)$ is increasing and concave down, and $f(0) = 1$, $f(10) = 7$, what can you say about $f(5)$? Justify your answer.

i. (*3 pts*) Consider the function $F(x) = 2(x+1)^3$. Find functions $f(x)$ and $g(x)$ so that $f(g(x)) = F(x)$.

j. (*3 pts*) What is the y-coordinate of the vertex of the parabola given by the equation $f(x) = ax^2 + bx + c$? (Your answer may contain some of the letters a, b, and c; you need not simplify your answer.)

5. (*20 points*) Circle **T** (True) or **F** (False), and fill in the blanks and boxes provided.

(**a**) **T** **F** All continuous functions are differentiable.

(**b**) **T** **F** All differentiable functions are continuous.

(**c**) **T** **F** $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$ represents the height of the function $f(x)$ at $x = 0$.

(**d**) **T** **F** A rational function can not have more than one horizontal asymptote.

(**e**) If f and g are continuous at $x = c$, then $f + g$ is ____________________ .

(**f**) If a and c are constants, then $\lim_{x \to a} \big(c \cdot h(x)\big) =$ ____________________ .

(**g**) If $\lim_{x \to \infty} f(x) = 2$, then $f(x)$ has a ____________________ at $y = 2$.

(continued)

(**h**) A function $f(x)$ is continuous at $x = 3$ if:

$$\lim_{\boxed{}} \boxed{} = \boxed{}.$$

(**i**) A function $f(x)$ is differentiable at $x = 3$ if:

$$\lim_{\boxed{}} \boxed{} \text{ exists.}$$

(**j**) If $f'(c) > 0$, then $f(x)$ is ____________________ at $x = c$.
(**k**) If $f''(c) > 0$, then $f(x)$ is ____________________ at $x = c$.
(**l**) If $f''(c) > 0$, then $f'(x)$ is ____________________ at $x = c$.

6. (*10 points*) Consider the following table of values for a function $f(x)$:

x	0	2	4	6	8	10
$f(x)$	0	30	52	68	80	88

(**a**) Approximate $f'(4)$ and $f'(6)$. Be sure to show your work.
(**b**) Use your answers to part (a) to approximate $f''(5)$. Again, show your calculations.

7. (*20 points*) Consider the function $f(x)$ defined by the following graph, and use the graph to find the quantities and answer the questions below.

(**a**) $\lim\limits_{x\to 1} f(x)$
(**b**) $\lim\limits_{x\to -3} f(x)$
(**c**) $\lim\limits_{x\to -1} f(x)$
(**d**) $\lim\limits_{h\to 0} \dfrac{f(-4+h) - f(-4)}{h}$
(**e**) $\lim\limits_{h\to -\infty} f(x)$

(continued)

(**f**) $\lim\limits_{h \to \infty} f(x)$

(**g**) The average rate of change of $f(x)$ from $x = -3$ to $x = -2$

(**h**) Where is $f(x)$ continuous?

(**i**) Where is $f(x)$ differentiable?

(**j**) Where is $f'(x) = 0$?

(**k**) Where is $f''(x) > 0$?

(**l**) Which quantity is greater, $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$ or $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$?

8. (*10 points*) Find a rational function that could have the graph below. Be sure to make it clear how you arrived at your function.

9. (*14 points*) Given the table below, find the value of $h'(3)$ for each given function $h(x)$. Showing work may earn you partial credit.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	2	4	5
2	5	1	3	4
3	4	5	2	1
4	1	3	5	2
5	2	4	1	3

(continued)

a. (*3 pts*) $h(x) = x \cdot g(f(x))$

b. (*3 pts*) $h(x) = (f(x))^2 + g(g(x))$

c. (*3 pts*) $h(x) = e^{f(x)+2g(x)}$

d. (*5 pts*) Which rules, *in order*, did you use in part (c) above?

10. (*14 points*) Consider the equation

$$x^3 + y^3 - xy^2 = 5.$$

a. (*5 pts*) Find $\dfrac{dy}{dx}$ by implicit differentiation.

b. (*5 pts*) Find the equation of the line tangent to the graph of $x^3 + y^3 - xy^2 = 5$ at the point $(1, 2)$.

c. (*4 pts*) Use the tangent line you found in part (b) to approximate the height of the graph of $x^3 + y^3 - xy^2 = 5$ when $x = 1.02$.

11. (*20 points*) Circle **T** (True) or **F** (False) or give short answers as appropriate. You may suppose that $f(x)$ is a continuous, differentiable function defined for all real numbers. (Parts (a)-(f) are worth 1 point each; parts (g)-(j) are worth 2 points each; parts (k) and (l) are worth 3 points each.)

(a) **T** **F** If $f''(a) = 0$ then $f(x)$ has an inflection point at $x = a$.

(b) **T** **F** Inflection points of $f(x)$ are the same thing as turning points of $f'(x)$.

(c) **T** **F** If $f'(a) = 0$ then $f''(a) = 0$.

(d) **T** **F** The derivative can only change sign at the critical points of $f(x)$.

(e) **T** **F** The function can only change concavity at the inflection points of $f(x)$.

(f) **T** **F** The global maximum of $f(x)$ is simply the highest local maximum.

(g) If $f'(x)$ has a local maximum at $x = a$, what can you say about $f(x)$ at $x = a$?

(h) If $x = a$ is an inflection point of $f(x)$, what can we say about $f''(x)$ at $x = a$?

(i) If $f(a) = f(b) = 0$, what can you say about $f'(x)$ on the interval $[a, b]$?

(j) If $f''(a) < 0$, what can you say about $f'(x)$ at $x = a$?

(continued)

(k) Suppose that $x = 4$ is a critical point of $f(x)$, and that $f''(4) = 0$. Is this enough information to tell if the critical point at $x = 4$ is a local maximum, a local minimum, or neither? If so, which is it? If not, why not?

(l) Suppose that $x = 4$ is a critical point of $f(x)$, and that $f'(0) > 0$ and $f'(5) < 0$. Is this enough information to tell if the critical point at $x = 4$ is a local maximum, a local minimum, or neither? If so, which is it? If not, why not?

12. (*10 points*) Use the definition of derivative to verify directly that the derivative of

$$f(x) = 2\sqrt{x+1} \quad \text{is} \quad f'(x) = \frac{1}{\sqrt{x+1}}.$$

(No credit will be given for using the "rules of derivatives.")

13. (*10 points*) You have \$5000 which you want to invest for 10 years in an IRA. You can choose from 2 accounts, one that pays 6% compounded annually and one that pays 5.5% compounded continuously. Which account would you choose?

14. (*15 points*) The population of 3-eyed fish in a lake next to a nuclear power plant was estimated at 10,000 when the plant was shut down. 2 years later the population was estimated at 6,400.

(a) Find a linear equation for the population of 3-eyed fish.

(b) Find an equation for the population of 3-eyed fish given exponential decay.

(c) What does the linear model predict for the population of fish 5 years after the plant shuts down? What about the exponential model?

(d) If the population is estimated at over 1000 10 years after the plant closed which model is better? Why?

15. (*12 points*) Let $f(x) = \ln \frac{2x+1}{x-3}$ be a function.

(a) Find the domain of $f(x)$. Do not use the calculator.

(b) Find the formula for its inverse function, $f^{-1}(x)$.

(c) Find: domain $f(x) =$ domain $f^{-1}(x) =$
range $f(x) =$ range $f^{-1}(x) =$

Show all your work and reasoning. Do not use calculator except for checking your answers.

(d) Find all x such that $f(x) = 0$.