Math 103.01 Summer 2001
Test #1

Name: ____________________________________

Read all of the following information before starting the test:

- Be sure that this test has **10** pages including this cover.
- There are **7** problems on this test worth a total of **100** points.
- The last page is for your scrap work and may be detached from the test booklet. The second last page entitled "Quadric Surfaces" may also be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	Total
Score								

1. (*20 points*) Suppose that $\mathbf{v}(t) = (-2\sin t, 2\cos t, 2)$ for $t \geq 0$ describes the *velocity* of a particle which starts at the point $(2, 0, 0)$.

a. (*4 pts*) Describe the parametric curve C traced out by this particle for $t \geq 0$.

b. (*12 pts*) Compute the Frenet frame for this particle when $t = \pi$. That is, determine the unit tangent vector $\mathbf{T}(\pi)$, the principal unit normal vector $\mathbf{N}(\pi)$, and the binormal vector $\mathbf{B}(\pi)$.

c. (*4 pts*) How far does this particle travel between $t = \pi$ and $t = 2\pi$? In other words, what is the arclength of the parametric curve C for $\pi \leq t \leq 2\pi$?

2. (*16 points*) Carefully show that each of the following limits do not exist.

a. (*8 pts*) $$\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4+y^6}$$

b. (*8 pts*) $$\lim_{(x,y,z)\to(0,0,0)} \frac{x^2+y^2}{x^2+y^2+z^2}$$

3. (*20 points*) Find parametric equations for the line through the point $(1, 8, 0)$ which is both parallel to the plane $x + y - 2z = 1$ and perpendicular to the line of intersection of the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

4. (*12 points*) Consider the surface $2x^3 - 6xy^2 + 3y^2 - 6z = 0$.

a. (*8 pts*) What is the equation of the tangent plane to the surface at the point $(3, 2, -1)$?

b. (*4 pts*) At which points is the tangent plane to the surface horizontal?

5. (*10 points*) Find all functions $f(x,y)$ which satisfy $\dfrac{\partial f}{\partial x} = 2\sin y + 3x^2y^2 + 6x^5$ and $\dfrac{\partial f}{\partial y} = 2x\cos y + 2x^3y + 5y^4$.

6. (*12 points*) Suppose that $f(x, y) = x^2y^2 + e^{xy} - \sin(x + y)$ where $x = \ln(rst)$ and $y = r + \cos(rst)$.

If $w = f(x, y)$, find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$, and $\frac{\partial w}{\partial t}$.

7. (*10 points*) Suppose that $f(x, y, z) = x^2 + y^2 - z^2$. Then the graph of $w = f(x, y, z)$ is a surface in the four dimensional space $\mathbb{R}^4$. Describe the traces of this surface in the coordinate planes $x = 0$, $y = 0$, $z = 0$, and $w = 0$.

Scrap Page

(*You may carefully remove this page from the test booklet.*)