Math 103.01 Summer 2001
Sample Test #1

1. (*20 points*) Let C be the parametrized path in $\mathbb{R}^3$ traced out by the position vector

$$\mathbf{r}(t) = \sin^3 t\,\mathbf{i} + \cos^3 t\,\mathbf{j}$$

for $0 \leq t \leq \pi/2$.

(**a**) Compute the arclength of C.

(**b**) Find the unit tangent and the unit normal to C for each t such that $0 \leq t \leq \pi/2$.

(**c**) Determine the equation the plane containing both the unit tangent and unit binormal vectors at $t = \pi/4$.

2. (*14 points*) Consider the function $f(x, y) = x^4 + y^4 - 4xy$.

(**a**) Find the equation of the plane tangent to the graph $z = f(x, y)$ at the point $(2, 2, 16)$.

(**b**) Find all points at which the tangent plane to the graph $z = f(x, y)$ is horizontal.

3. (*14 points*) Find an equation of the plane that passes through the point $(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

4. (*10 points*) Give a geometric description of the family of planes $x + y + z = k$ where k is a real constant.

5. (*20 points*) Consider the following limits. For each, either show that the limit does not exist or determine its value if it does.

(**a**) $$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$$

(**b**) $$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$

6. (*10 points*) Suppose that $f(x, y) = x^3 + \alpha xy^2$. Find the value of α so that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

7. (*12 points*) If $z = f(x - y)$, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.