Math 103.01 Summer 2001
Bonus Test
Due: Wednesday, June 20, 2001

The following problems are worth bonus points on test #2. You must work through all problems on your own. You may consult any reference materials but do not discuss these problems with anyone else in the class. Show all work neatly and in order, and clearly indicate your final answers.

1. (*4 points*) Suppose that T is the region in the first octant which lies between the spheres $x^2 + y^2 + z^2 = 2$ and $x^2 + y^2 + z^2 = 4$. That is, T is defined by the inequalities $x \geq 0$, $y \geq 0$, $z \geq 0$, and $2 \leq x^2 + y^2 + z^2 \leq 4$. Evaluate

$$\int\int\int_T \frac{z}{x^2 + y^2 + z^2}\, dV.$$

2. (*4 points*) As you may know, the area of the unit disk $x^2 + y^2 = 1$ in $\mathbb{R}^2$ is π and the volume of the unit sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$ is $\frac{4}{3}\pi$.

There is a natural generalization to 4 dimensions. The surface $x^2 + y^2 + z^2 + w^2 = 1$ is the called the unit hypersphere in $\mathbb{R}^4$.

Write down a quadruple integral to express the hypervolume of the unit hypersphere in $\mathbb{R}^4$.

Show that the volume of the unit hypersphere in $\mathbb{R}^4$ is $\frac{\pi^2}{2}$ by explicitly evaluating the inner integral and then changing the region described by the outer 3 integrals to spherical coordinates.