Math 103.01 Summer 2001
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Fundamental Theorems of Vector Calculus

Green's Theorem: Suppose that $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ is a vector field in the plane and that R is a bounded region in $\mathbb{R}^2$ with smooth boundary C. Suppose further that $\mathbf{T}$ is a unit tangent to C which is positively oriented. Then

$$\oint_C \mathbf{F} \cdot \mathbf{T}\, ds = \iint\limits_R \nabla \times \mathbf{F}\, dA.$$

Note: Green's theorem may also be written as

$$\oint_C P\, dx + Q\, dy = \iint\limits_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

Stokes' Theorem: Suppose that S is a surface in $\mathbb{R}^3$ with boundary C and unit normal $\mathbf{n}$. Also suppose that $\mathbf{T}$ is the unit tangent field along C such that $\mathbf{n} \times \mathbf{T}$ points into S. Then,

$$\oint_C \mathbf{F} \cdot \mathbf{T}\, ds \iint\limits_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS.$$

Note: Stokes' theorem may also be written as

$$\oint_C P\, dx + Q\, dy + R\, dz = \iint\limits_S \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) dydz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) dzdx + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy.$$