Math 103.01 Summer 2001
Assignment #5 Solutions

1.

a. div $\mathbf{F} = 2xz^3 + 6xy^2z$

b. $\mathbf{F}$ is conservative since $F = \nabla f$ where $f(x, y, z) = xy^2z^3$. Thus curl $\mathbf{F} = 0$.

c. Since $\mathbf{F}$ is conservative and C is a closed curve, the fundamental theorem gives $\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C \nabla f \cdot d\mathbf{r} = 0$.

2. By Green's theorem, $\oint_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \int_0^{x^2} (2x + 2y)\ dydx = 2/5$.

3.

a. $\frac{5\sqrt{5} - 1}{12}, \frac{1}{2}, \frac{2}{3}$

b. Parametrize C: $x = t$, $y = t^2$, $-1 \le t \le 2$. Then, $\oint_C xy\ dx + (x + y)\ dy = \int_{-1}^{2} (3t^3 + 2t^2)\ dt = \frac{69}{4}$.

c. $\oint_C \mathbf{F} \cdot \mathbf{T}\ ds = \oint_C y\ dx - x\ dy + z\ dz = \int_0^{\pi} (\cos^2 t + \sin^2 t + 4t)\ dt = \pi + 2\pi^2$.

4.

a. Conservative: $f(x, y) = x^2y^2 + x^3 + y^4$

b. Not conservative: Since $\frac{\partial P}{\partial y} = -x \sin y + \cos y$ and $\frac{\partial Q}{\partial x} = -y \sin x + \cos x$ and they are not equal, $\mathbf{F}$ is not conservative.

c. $f(x, y, z) = xyz + \frac{1}{2}y^2 + z$

5.

a. Odd answers are in the back of the book. Let me know if you'd like to see a specific even number.

b. Using the given parametrization, $A = \oint_C x\ dy = \oint_C (a \cos t)(a \sin t)\ dt = a^2 \int_0^{2\pi} \frac{1}{2}(1 + \cos 2t)\ dt = \pi a^2$.

c. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2y - 2y = 0$ so $W = 0$.