Math 171 Prelim #2 – April 13, 2004

Malott Hall, Room 251: Section 01 (Bendikov) & Section 02 (Kozdron)
Malott Hall, Room 253: Section 03 (Hwang) & Section 04 (Hwang)

*You have **90** minutes to complete this exam. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.*

Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct. *Points will be deducted for incoherent, incorrect, and/or irrelevant statements. Calculators are permitted, as well as one handwritten* 8.5×11 *page of notes (double-sided). No other aids are allowed, but a table of normal probabilities will be provided.*

You are allowed to use standard notation. However, any new notation or abbreviations that you introduce must be clearly defined.

*This examination consists of **6** problems and is worth **100** total points. (Each sub-part is worth 5 points.)*

All work must be completed in the examination booklets provided.

Good luck!

1. (*15 points*) Consider the following two populations:

Population 1 has a mean of 17 and a variance of 144.
Population 2 has a mean of 18 and a variance of 400.

Suppose that we select a simple random sample of size 16 from Population 1, and write $\overline{X}$ for this sample average. Independently, we select a simple random sample of size 25 from Population 2, and write $\overline{Y}$ for this sample average. For each of the following random variables, compute its mean and standard deviation. *(You must show your work. It is not enough to simply write down the answers.)*

(a) $\overline{X}$

(b) $\overline{Y}$

(c) $\overline{X} - \overline{Y}$

2. (*10 points*) The *Finger Lakes Fruit Company* claims that the weights of the apples they supply is normally distributed with mean 5 oz. and standard deviation 0.3 oz. They also claim that the weights of the oranges they supply is normally distributed with mean 6 oz. and standard deviation 0.4 oz.

(a) Suppose that one apple and one orange are selected at random. What is the probability that the apple is heavier than the orange?

(b) Suppose that 3 apples and 4 oranges are selected at random. What is the probability that the average weight of the 3 apples is more than the average weight of the 4 oranges?

3. (*10 points*) Researchers at the *Finger Lakes Medical Center* suspect that added calcium in diet reduces blood pressure. They have selected a group of volunteers named

{ Abraham, Beth, Charles, Danielle, Edward, Francine, Gene, Holly, Isaiah, Jessica }

Use the following extract of *Table B: Random Digits* to answer the questions below:

96767 35964 23822 96012 94591 65194 50842 53372 (Line 141)

(a) The researchers will conduct an experiment to test if added calcium in diet does reduce blood pressure. They have decided that the experiment will include a treatment group and a placebo group (that is, a control group). Randomly assign five people to the treatment group and the other five to the placebo group. Be sure to carefully explain your method of random allocation, and write the names of the volunteers in the treatment group.

(b) Describe how the researchers could conduct this experiment so that it is double blind.

4. (*15 points*) A Cornell entomologist measured the length and age of 51 worms found in the *Finger Lakes National Forest*. A summary of the data she obtained is as follows:

	Length (mm)	Age (months)
mean	24	23
SD	1.5	2.2

She also found that $\sum(x_i - \overline{x})(y_i - \overline{y}) = 121$, where the variable x represents length, and the variable y represents age.

(a) Find the correlation between the age and length of the worms in this study.

(b) The entomologist wants to express age in terms of length. Find the equation of the regression line ($\hat{y} = a + bx$) for age in terms of length.

(c) Suppose that a fisherman is walking in the forest looking for worms for bait, and he randomly selects a worm that is 28.3 mm long. Using your answer to **(b)**, what is the predicted age of this worm?

5. (*25 points*) Customer satisfaction surveys collected by the *Finger Lakes Motor Company* show that owners of the *Cortland* car have gas mileages that are normally distributed with mean $\mu = 33.4$ and standard deviation $\sigma = 2.6$ (miles per gallon). The *Finger Lakes Motor Company* also makes the *Elmira* car which is identical except for a new fuel injection system. Ten owners of the *Elmira* reported their gas mileages (in miles per gallon) as shown below:

36	35	41	36	33
33	39	32	38	51

(a) Construct a 95% confidence interval for the mean gas mileage for the *Elmira*. (Assume the population standard deviation is 2.6.)

(b) Explain in context what is meant by "95% confidence."

(c) The company claims that the new fuel injection system appears to have an effect on gas mileage. Carefully define your notation, and write down the appropriate null hypothesis H_0 and the alternative hypothesis H_a necessary to conduct a test of the company's claim.

(d) Based on your confidence interval from **(a)**, can you reject the null hyposthesis H_0 from **(b)** at the $\alpha = 0.05$ level? (Assume the population standard deviation is 2.6.)

(e) Suppose you need a better estimate of the *Elmira*'s gas mileage. How many customer satisfaction surveys must be collected in order to have 98% confidence of estimating the true mean to within 0.5? (Assume the population standard deviation is 2.6.)

6. (*25 points*) Carla is a pizza inspector for the *Finger Lakes Health Authority.* She has received numerous complaints about a certain pizzeria for failing to comply with its advertisements. The pizzeria claims, on the average, that each of its pizzas is topped with 4 oz. of pepperoni. The dissatisfied customers feel that the actual amount of pepperoni used is considerably less. To settle the matter, Carla decides to do a hypothesis test. She assumes that the distribution of pepperoni weights on a pizza is normal with standard deviation $\sigma = 0.5$ oz. and wants to draw conclusions about the mean weight μ. Her objective is to test $H_0 : \mu = 4$ versus $H_a : \mu < 4$. For data, she collects 4 large pizzas and finds the mean weight of pepperoni. If the mean weight is less than 3.6 oz. she will reject H_0.

- **(a)** Compute the probability of a Type I error for this test.
- **(b)** Compute the probability of a Type II error for this test assuming that the true mean is $\mu = 3.5$.
- **(c)** What decision rule would have significance level $\alpha = 0.01$?
- **(d)** Compute the probability of a Type II error for the decision rule in **(c)** assuming $\mu = 3.5$.
- **(e)** In the context of this problem, if Carla should present her findings to a court, should both Type I and Type II errors carry equal weight?