Math 171.04 Spring 2004
February 23, 2004
Topics Covered for Prelim #1

Course packet: 4.1, 4.2, 4.3, 4.4, 4.5

-sample space, events, probability

-union and intersection; complement; Venn diagrams

-properties of probability

-sum rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

-disjoint events: $A \cap B = \emptyset$, or equivalently, $P(A \cap B) = 0$

-conditional probability: $P(A \cap B) = P(A|B) \cdot P(B)$ always

-Since $A \cap B = B \cap A$ we have the equivalence: $P(A \cap B) = P(A|B) \cdot P(B)$ and $P(B \cap A) = P(B|A) \cdot P(A)$ so that

$$P(A|B) \cdot P(B) = P(B|A) \cdot P(A).$$

This is useful for finding one conditional probability when only the other conditional probability is given.

-independent events: A and B are independent if $P(A|B) = P(A)$, or equivalently, if $P(A \cap B) = P(A) \cdot P(B)$

Course packet: 5.1, 5.2, 5.3, 5.4, 5.5

-random variables (discrete)

-distribution of random variable: listing of all possible probability assignments $P(X = i)$

-expected value $E(X)$: measures mean, or weighted average of a random variable. Know how to compute it: $E(X) = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$.

-For any random variables, we have $E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$, and for any constant c we have $E(cX) = c\,E(X)$ and $E(X + c) = E(X) + c$.

-variance Var(X): measure spread of a random variable. Know how to compute it: Var$(X) = E(X^2) - (E(X))^2$.

-We always have Var$(cX) = c^2$Var(X) and Var$(X + c)$ =Var(X) for any constant c. If the random variables $X_1, X_2, \ldots, X_n$ are independent, then

$$\text{Var}(X_1 + X_2 + \cdots + X_n) = \text{Var}(X_1) + \text{Var}(X_2) + \cdots + \text{Var}(X_n).$$

In particular, if X and Y are independent, then Var$(X + Y)$ =Var$(X - Y)$ =Var(X)+Var(Y).

-Standard deviation is the square root of the variance. Hint: always compute with variances, and take square roots at the end of the problem. SD measures the expected deviation a random variable is from its mean.

-Bernoulli random variable: X is Bernoulli with parameter p if $P(X = 1) = p$ and $P(X = 0) = 1-p$. A Bernoulli random variable counts a success on a single trial where there is probability p of success. Compute: $E(X) = p$, Var$(X) = p(1 - p)$.

-Binomial random variable: X is Binomial with parameters n and p if

$$P(X = i) = \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i}.$$

Note, a binomial random variable counts the total number of successes in n independent trials where there is probability p of success on each trial. Compute: $E(X) = np$, Var$(X) = np(1 - p)$.

Normal random variables: Chapter 3

-If Z is normal with mean 1 and SD 0, then corresponding probabilities can be computed by finding the area under the normal density curve. This is exactly what Table A in the textbook is for.

-If X is normal with mean μ and SD σ, then $\frac{X-\mu}{\sigma}$ is normally distributed with mean 1 and SD 0. CONSEQUENCE: no matter what normal distribution you are given, you can find corresponding probabilities by *normalizing* (a.k.a. finding *z-scores*) and using the table.

-If Z is normal with mean 1 and SD 0, then $X = \sigma Z + \mu$ is normal with mean μ and SD σ. This is useful when *reading the table backwards* (a.k.a. finding *inverse z-scores*).

-If X_1, $X_2, \ldots, X_n$ are independent and identically distributed (iid) normal with mean μ and SD σ, then $\overline{X} = \frac{X_1+X_2+\cdots+X_n}{n}$ is normally distributed with mean μ and SD $\frac{\sigma}{\sqrt{n}}$. This is a special case of the central limit theorem.

-This fact is useful for computing probabilities associated with the *sum* of iid normal random variables.

Normal Approximation to the Binomial: Chapter 12 (pages 312–314)

-If X is binomial with parameters n, p, then $E(X) = np$, and $\mathrm{Var}(X) = np(1-p)$. As another special case of the central limit theorem, if n is large and p is not near 0 or 1, then the distribution of X is *approximately* normal with mean np and SD $\sqrt{np(1-p)}$. This is useful for doing calculations because normal probabilities are easily computed from the table.

-Example: What is the probability of flipping exactly 47 heads on 100 tosses of a fair coin? (a) write out the exact solution using a binomial $n = 100$, $p = 1/2$ with 47 success. The problem is when you try to compute this. (b) find an approximate solution by finding the probability that a normal mean $100 \times 1/2$, SD $\sqrt{100 \times 1/2 \times 1/2}$ is between 46.5 and 47.5.

Sample Proportions: Chapter 18 (pages 469–473)

-If $\hat{p}$ is the observed proportion (sample proportion) of some characteristic, where there are n total trials and probability p of success on each trial, then the sampling distribution of $\hat{p}$ is approximately normal with mean p and SD $\sqrt{\frac{p(1-p)}{n}}$.

-Again, this is useful for doing calculations from Table A.